

# A General Pre-FFT Criterion for MIMO-OFDM Beamforming

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**Abstract**—In this paper, we propose a general beamforming criterion for pre-FFT processing in orthogonal frequency division multiplexing (OFDM) systems with multiple transmit and receive antennas. The proposed criterion depends on a single parameter  $\alpha$ , which establishes a tradeoff between the energy of the equivalent SISO channel (after Tx-Rx beamforming) and its spectral flatness. The proposed cost function embraces most reasonable criteria for designing Tx-Rx pre-FFT beamformers. Hence, for particular values of  $\alpha$  the proposed criterion reduces to the minimization of the mean square error (MSE), the maximization of the system capacity, or the maximization of the received signal-to-noise ratio (SNR). In general, the proposed criterion results in a non-convex optimization problem. However, we show that the problem can be approximately solved by semidefinite relaxation (SDR) techniques. Additionally, since the computational cost of SDR for this problem is rather high, we propose a simple yet efficient gradient search algorithm which provides satisfactory solutions with a moderate computational cost for OFDM-based WLAN standards such as 802.11a. Finally, the good performance of the proposed technique is illustrated by means of some numerical results.

## I. INTRODUCTION

In the last years, pre-FFT beamforming schemes have emerged as an interesting alternative for exploiting spatial diversity in multiple-input multiple-output (MIMO) systems under orthogonal frequency division multiplexing (OFDM) transmissions [1]–[3]. The key idea behind pre-FFT processing consists in combining the received (respectively transmitted) signals before (resp. after) FFT processing, thus avoiding the necessity of one FFT block for each antenna (see Fig. 1). An equivalent scheme appears when beamforming is performed in the analog domain [4], [5].

The price to be paid by this reduction in the system complexity is a slight performance degradation in comparison to a full MIMO system that uses optimal per-carrier processing. Obviously, this limitation comes from the fact that the same pair of transmit/receive beamformers is applied to all the subcarriers.

The research leading to these results has received funding from the European Community's Seventh Framework Programme (FP7/2007-2013) under grant agreement n 213952, MIMAX. Additionally, the work of the first three authors has also been supported by the Spanish Government (MICINN) under projects TEC2007-68020-C04-02/TCM (MULTIMIMO) and CONSOLIDER-INGENIO 2010 CSD2008-00010 (COMONSENS).

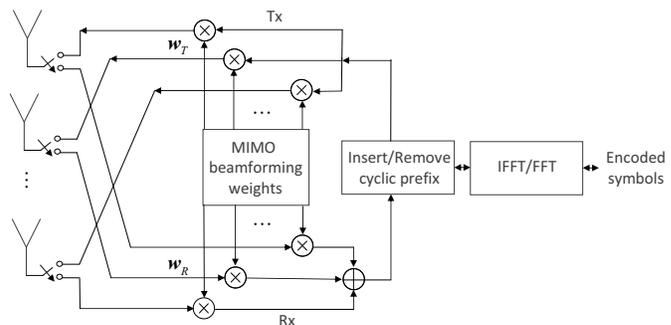


Fig. 1. Pre-FFT MIMO beamforming scheme.

Previous works have considered the problem of designing the pre-FFT beamformers in order to maximize the received SNR [1]–[4], and more recently, we have proposed two new beamforming criteria based on the maximization of the system capacity [6], and the minimization of the bit error rate (BER) of the optimal linear receiver [7]. In this work, we propose a general pre-FFT beamforming criterion which includes, as particular cases, all the previous approaches. The proposed cost function is inspired by the Renyi entropy definition, and it depends on a single parameter  $\alpha$ , which establishes a tradeoff between the energy of the equivalent channel (after Tx-Rx beamforming) and its spectral flatness. Thus, for particular values of this parameter, the proposed criterion reduces to the maximization of the SNR (MaxSNR,  $\alpha = 0$ ), the maximization of the system capacity (MaxCAP,  $\alpha = 1$ ), the minimization of the MSE of the optimal linear receiver (MinMSE,  $\alpha = 2$ ), and the optimization of the worst subcarrier response (MaxMin,  $\alpha = \infty$ ).

Unfortunately, the proposed criterion results in a non-convex optimization problem. However, we show that approximated solutions can be obtained by means of semidefinite relaxation (SDR) techniques. Furthermore, in order to avoid the high computational cost of SDR approaches for practical problems, we propose a simple iterative algorithm which, equipped with a good initialization method, provides satisfactory results in most practical cases.

## II. DATA MODEL

Let us consider a pre-FFT MIMO system with  $n_T$  transmit and  $n_R$  receive antennas, and with unit-energy transmit and receive beamformers. Assuming a cyclic prefix longer than the channel impulse response, the communication system after Tx-Rx beamforming may be decomposed into the following set of parallel and non-interfering single-input single-output (SISO) equivalent channels

$$y_k = h_k s_k + n_k, \quad k = 1, \dots, N_c, \quad (1)$$

where  $N_c$  is the number of data carriers,  $y_k \in \mathbb{C}$  is the observation associated to the  $k$ -th data carrier,  $n_k$  represents the complex circular i.i.d. Gaussian noise with zero mean and variance  $\sigma^2$ ,  $s_k$  is the transmitted signal, and  $h_k$  is the frequency response of the equivalent channel after Tx-Rx beamforming, which is given by

$$h_k = \mathbf{w}_R^H \mathbf{H}_k \mathbf{w}_T, \quad k = 1, \dots, N_c, \quad (2)$$

where  $\mathbf{w}_T \in \mathbb{C}^{n_T \times 1}$  and  $\mathbf{w}_R \in \mathbb{C}^{n_R \times 1}$  are the transmit and receive beamformers, and  $\mathbf{H}_k \in \mathbb{C}^{n_R \times n_T}$  represents the response of the MIMO channel for the  $k$ -th data-carrier.

Under perfect knowledge of the equivalent channel and noise variance, and assuming unit transmit power per data carrier ( $E[|s_k|^2] = 1$ ), the linear minimum mean square error (LMMSE) estimate of  $s_k$  is  $\hat{s}_k = \frac{h_k^* y_k}{|h_k|^2 + \sigma^2}$ , which yields a per-carrier MSE

$$\text{MSE}_k = E[|\hat{s}_k - s_k|^2] = \frac{1}{1 + \gamma |h_k|^2}, \quad k = 1, \dots, N_c, \quad (3)$$

where  $\gamma = 1/\sigma^2$  is defined as the (expected) signal to noise ratio (SNR) at the transmitter side.

## III. GENERAL PRE-FFT MIMO BEAMFORMING CRITERION

In this section we introduce a general criterion for the design of the Tx-Rx beamformers under perfect knowledge of the MIMO channel  $\mathbf{H}_k$  and noise variance at the receiver side.<sup>1</sup> Specifically, we propose to minimize the following cost function

$$f_\alpha(\mathbf{w}_T, \mathbf{w}_R) = \frac{1}{\alpha - 1} \log \left( \frac{1}{N_c} \sum_{k=1}^{N_c} \text{MSE}_k^{\alpha-1} \right), \quad 0 \leq \alpha < \infty \quad (4)$$

which is inspired by the definition of Renyi's entropy of order  $\alpha$  for a discrete random variable [9], [10]. Thus, our optimization problem can be written as

$$\arg \min_{\mathbf{w}_T, \mathbf{w}_R} f_\alpha(\mathbf{w}_T, \mathbf{w}_R) \quad \text{subject to} \quad \|\mathbf{w}_T\| = \|\mathbf{w}_R\| = 1, \quad (5)$$

which encompasses the following interesting beamformer design criteria:

- MaxSNR ( $\alpha = 0$ ): If the parameter  $\alpha$  is set to zero, the optimization problem in (5) reduces to the maximization

<sup>1</sup>Some numerical results including the estimation of the channel and noise variance can be found in the journal version of this paper [8].

of the energy of the equivalent channel, or equivalently, to the maximization of the received SNR. [1]–[4].

- MaxCAP ( $\alpha = 1$ ): When  $\alpha$  approaches 1, it can be easily shown by direct application of the L'Hopital's rule, that the proposed criterion reduces to the maximization of the system capacity [6].
- MinMSE ( $\alpha = 2$ ): In this case, (5) is equivalent to the minimization of the overall MSE of the optimal linear receiver [7].
- MaxMin ( $\alpha = \infty$ ): In this case, the summation in (4) is dominated by the worst data-carrier, i.e., by that with the smallest  $|h_k|^2$ . Therefore, for  $\alpha \rightarrow \infty$ , the proposed criterion reduces to the optimization of the worst data-carrier.

### A. Cost Function Properties

In this subsection, we briefly review the main properties of the proposed beamforming criterion. For further details, as well as a rigorous proof of the properties, we refer to [8].

*Property 1:* In the low SNR regime ( $\gamma \rightarrow 0$ ), the proposed criterion reduces to the MaxSNR approach regardless of  $\alpha$ .

*Property 2:* The solutions ( $\tilde{\mathbf{w}}_T, \tilde{\mathbf{w}}_R$ ) of the proposed beamforming criterion are Pareto optimal points of the multi-objective optimization problem based on the individual channel energies ( $|h_k|^2$ ) or MSEs ( $\text{MSE}_k$ ).

Property 1 ensures the equivalence of all the criteria in the low SNR regime. Property 2 implies that, given the optimal pair of beamformers ( $\tilde{\mathbf{w}}_T, \tilde{\mathbf{w}}_R$ ) for some value of  $\alpha$ , then there does not exist another feasible pair ( $\mathbf{w}_T, \mathbf{w}_R$ ) satisfying

$$|\mathbf{w}_R^H \mathbf{H}_k \mathbf{w}_T|^2 \geq |\tilde{\mathbf{w}}_R^H \mathbf{H}_k \tilde{\mathbf{w}}_T|^2, \quad k = 1, \dots, N_c, \quad (6)$$

with at least one strict inequality. However, we must note that the converse of Property 2 is not true in general. This fact is illustrated by means of a toy example consisting of a SIMO system with  $N_c = 2$  data carriers,  $n_R = 2$  receive antennas, real beamformers and subcarrier channels given by  $\mathbf{H}_1 = [1, 0]^T$  and  $\mathbf{H}_2 = [0.35, -0.8]^T$ .

Fig. 2 shows the set of achievable points ( $|h_1|^2, |h_2|^2$ ) with unit energy beamformers  $\mathbf{w}_R$ , where we can see that the solutions of the proposed criterion for different values of  $\alpha$  are only a subset of the Pareto optimal solutions. However, it should be pointed out that those points in the Pareto boundary that are not achievable by our criterion might not be of practical interest. As an example, consider the Pareto points in Fig. 2 near  $|h_1|^2 = 0.4$ ,  $|h_2|^2 = 0.7$ . Clearly, these points are worse solutions than those near  $|h_1|^2 = 0.8$ ,  $|h_2|^2 = 0.4$ , which are solutions of the proposed criterion for some  $\alpha \simeq 0$ .

The analogy of the proposed cost function with Renyi's entropy allows us to shed some light into the effect of the parameter  $\alpha$ . Specifically, when  $\alpha$  increases from 0 to  $\infty$ , the proposed beamforming criterion tends to flatten the equivalent channel, i.e., the critical data carriers (those with the smallest  $|h_k|$ ) are improved at the expense of the rest. In other words, we are sacrificing part of the channel energy in order to obtain less frequency selective equivalent SISO channels.

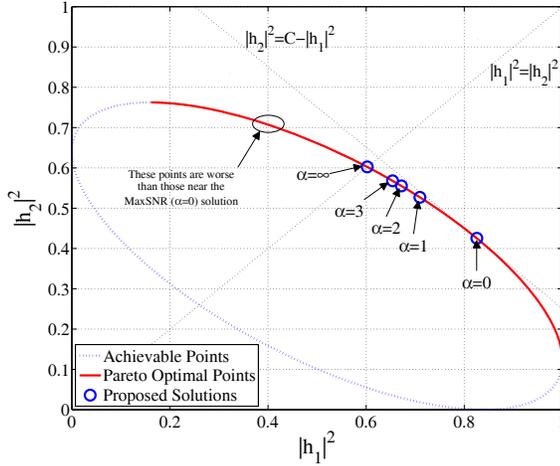


Fig. 2. Toy example with a  $1 \times 2$  SIMO system with  $N_c = 2$  subcarriers. The figure shows the set of achievable points with  $\|\mathbf{w}_R\| = 1$  (dotted line), the Pareto optimal points (solid line), and the solutions associated to the proposed criterion (the section of the curve marked with circles).

TABLE I  
PARTICULAR CASES WITH CLOSED-FORM SOLUTIONS.

System	$\mathbf{w}_T$	$\mathbf{w}_R$
SIMO with $\alpha = 0$ or $\gamma \simeq 0$	1	MRC
MISO with $\alpha = 0$ or $\gamma \simeq 0$	MRT	1
MIMO flat fading	MRT	MRC

#### IV. OPTIMIZATION PROBLEM AND PROPOSED ALGORITHM

Although the optimization problem (5) is in general non-convex, in this section we show that approximate solutions can be obtained by means of semidefinite relaxation (SDR) techniques. However, the computational cost associated to SDR techniques can be very high even for a moderate number of data subcarriers and antennas. For this reason, we also propose a simple gradient search method which is initialized using a closed-form approximation of the MaxSNR solution. As it will be shown in Section V, this method provides very accurate results.

##### A. Optimization Problem

Let us start by rewriting the optimization problem in (5) as

$$\arg \min_{\mathbf{W}} \frac{1}{\alpha - 1} \log \left( \frac{1}{N_c} \sum_{k=1}^{N_c} \left( 1 + \gamma |\text{Tr}(\mathbf{H}_k \mathbf{W})|^2 \right)^{1-\alpha} \right), \quad (7)$$

subject to  $\|\mathbf{W}\| = 1$ ,  $\text{rank}(\mathbf{W}) = 1$ ,

where  $\mathbf{W} = \mathbf{w}_T \mathbf{w}_R^H$  is the rank-one Tx-Rx beamforming matrix. Although the solution of the above problem can be obtained in closed-form in some particular cases (see Table I), in general this is a very difficult problem due to the rank-one constraint and the non-convexity of the cost function, which precludes the application of standard convex optimization techniques [11]. Nevertheless, we can gain some insight by

applying the Lagrange multipliers method. In particular, it is easy to prove that the local minima of (7) are also solutions of the following coupled eigenvalue (EV) problems

$$\mathbf{R}_{\text{MISO}_\alpha} \mathbf{w}_T = \lambda \mathbf{w}_T, \quad \mathbf{R}_{\text{SIMO}_\alpha} \mathbf{w}_R = \lambda \mathbf{w}_R, \quad (8)$$

where  $\lambda = \sum_{k=1}^{N_c} \text{MSE}_k^\alpha |h_k|^2$ ,

$$\mathbf{R}_{\text{MISO}_\alpha} = \sum_{k=1}^{N_c} \text{MSE}_k^\alpha \mathbf{h}_{\text{MISO}_k} \mathbf{h}_{\text{MISO}_k}^H, \quad (9)$$

$$\mathbf{R}_{\text{SIMO}_\alpha} = \sum_{k=1}^{N_c} \text{MSE}_k^\alpha \mathbf{h}_{\text{SIMO}_k} \mathbf{h}_{\text{SIMO}_k}^H, \quad (10)$$

can be seen as weighted covariance matrices, and

$$\mathbf{h}_{\text{MISO}_k} = \mathbf{H}_k^H \mathbf{w}_R, \quad \mathbf{h}_{\text{SIMO}_k} = \mathbf{H}_k \mathbf{w}_T, \quad (11)$$

are the MISO (SIMO) channels after fixing the receive (transmit) beamformer. Thus, the proposed beamforming criterion tries to maximize a weighted-sum of the energies of the equivalent SISO channel, where the weights are precisely given by  $\text{MSE}_k^\alpha$  and hence the higher weights are given to the worst subcarriers.

##### B. Approximated Solution based on Semidefinite Relaxation

In order to obtain an approximate solution to our non-convex optimization problem, it has to be reformulated in a form suitable for the application of semidefinite relaxation (SDR) techniques. Thus, it is not difficult to prove that (7) can be rewritten as [8]

$$\begin{aligned} \arg \min_{\mathbf{W}, \tilde{\mathbf{W}}, \gamma_1, \dots, \gamma_{N_c}} & \frac{1}{\alpha - 1} \sum_{k=1}^{N_c} (1 + \gamma_k)^{1-\alpha}, \\ \text{subject to} & \gamma \text{Tr}(\tilde{\mathbf{H}}_k \tilde{\mathbf{W}}) = \gamma_k, \quad k = 1, \dots, N_c \\ & \text{Tr}(\tilde{\mathbf{W}}) = 1, \quad \tilde{\mathbf{W}} \succeq \mathbf{0}, \\ & \text{vec}(\mathbf{W}) = \mathbf{v}_{\max}(\tilde{\mathbf{W}}), \\ & \text{rank}(\tilde{\mathbf{W}}) = 1, \quad \text{rank}(\mathbf{W}) = 1, \end{aligned} \quad (12)$$

where  $\tilde{\mathbf{H}}_k = \mathbf{h}_k \mathbf{h}_k^H$ ,  $\mathbf{h}_k = \text{vec}(\mathbf{H}_k^H)$  can be seen as a virtual SIMO or MISO channel with  $n_T n_R$  antennas,  $\mathbf{w} = \text{vec}(\mathbf{W})$ , and  $\tilde{\mathbf{W}} = \mathbf{w} \mathbf{w}^H$  is the associated  $n_T n_R \times n_T n_R$  rank-one beamforming matrix. On the other hand,  $\mathbf{v}_{\max}(\tilde{\mathbf{W}})$  denotes the eigenvector corresponding to the maximum eigenvalue of  $\tilde{\mathbf{W}}$ , and  $\tilde{\mathbf{W}} \succeq \mathbf{0}$  means that  $\tilde{\mathbf{W}}$  is hermitian and semidefinite positive.

Now, dropping the two rank-one constraints we obtain the following convex optimization problem<sup>2</sup>

$$\begin{aligned} \arg \min_{\tilde{\mathbf{W}}, \gamma_1, \dots, \gamma_{N_c}} & \frac{1}{\alpha - 1} \sum_{k=1}^{N_c} (1 + \gamma_k)^{1-\alpha}, \\ \text{subject to} & \gamma \text{Tr}(\tilde{\mathbf{H}}_k \tilde{\mathbf{W}}) = \gamma_k, \quad k = 1, \dots, N_c \\ & \text{Tr}(\tilde{\mathbf{W}}) = 1, \quad \tilde{\mathbf{W}} \succeq \mathbf{0}, \end{aligned} \quad (13)$$

<sup>2</sup>Note that the matrix  $\mathbf{W}$  can be removed because it only appears in the constraint  $\text{vec}(\mathbf{W}) = \mathbf{v}_{\max}(\tilde{\mathbf{W}})$ .

which can be solved by means of standard convex optimization techniques. Finally, an approximated rank-one solution to (12) can be generated from the (in general full-rank) solution of (13) by means of randomization [12] or a similar approach.

In general the computational complexity of the SDR technique can be prohibitive for practical applications. As an example, let us consider a practical MIMO system such as that used in the simulations ( $N_c = 64$  data carriers and  $n_T = n_R = 4$  antennas). In this case, we have  $N_c = 64$  slack variables  $\gamma_k$  and a  $16 \times 16$  semidefinite positive matrix  $\tilde{\mathbf{W}}$ . Thus, even assuming a linear cost function, the computational cost of the problem in (13) can be as large as  $\mathcal{O}((N_c + n_T^2 n_R^2)^{3.5}) \approx 5 \cdot 10^8$  [12]. Furthermore, the application of a randomization method with a sufficient number of candidates (see [12] for typical numbers of randomizations used in a moderate-sized problem) would also result in a prohibitive computational burden.

### C. Proposed Beamforming Algorithm

In order to avoid the high computational complexity associated to the SDR approach, we propose a simple iterative algorithm which, equipped with an adequate initialization point, provides good results in most practical cases. The proposed algorithm can be seen as a generalization of those proposed in [6], [7] for the MaxCAP ( $\alpha = 1$ ) and MinMSE ( $\alpha = 2$ ) cases. Let us start by briefly describing the initialization method, which obtains an approximated MaxSNR solution in closed form. In particular, for  $\alpha = 0$  (or  $\gamma \simeq 0$ ) the optimization problem in (7) can be rewritten as

$$\begin{aligned} \arg \max_{\mathbf{W}, \mathbf{w}} \quad & \sum_{k=1}^{N_c} \mathbf{w}^H \tilde{\mathbf{H}}_k \mathbf{w}, \\ \text{subject to} \quad & \|\mathbf{w}\| = 1, \\ & \text{vec}(\mathbf{W}) = \mathbf{w}, \quad \text{rank}(\mathbf{W}) = 1. \end{aligned} \quad (14)$$

Thus, defining  $\mathbf{R} = \sum_{k=1}^{N_c} \tilde{\mathbf{H}}_k$  and relaxing the rank-one constraint, we can obtain an approximate solution of the above problem as the principal eigenvector of  $\mathbf{R}$ . As previously pointed out, the matrix  $\mathbf{W}$  obtained from the solution  $\mathbf{w} = \mathbf{v}_{\max}(\mathbf{R})$  will not be rank-one in general. Here, we propose to obtain the best (in the squared-norm sense) rank-one approximation of  $\mathbf{W}$ , i.e., the transmit and receive beamformers are given by the left and right singular vectors of  $\mathbf{W}$ .

After obtaining the initialization point, the proposed gradient search algorithm is based on the following updating rules

$$\mathbf{w}_T(t+1) = \mathbf{w}_T(t) + \mu \mathbf{R}_{\text{MISO}_\alpha}(t) \mathbf{w}_T(t), \quad (15)$$

$$\mathbf{w}_R(t+1) = \mathbf{w}_R(t) + \mu \mathbf{R}_{\text{SIMO}_\alpha}(t) \mathbf{w}_R(t), \quad (16)$$

where  $\mu$  is a step-size (or regularization parameter) and  $t$  denotes the iteration index.<sup>3</sup> The above expressions can be also interpreted as iterations of a (regularized) power method for

<sup>3</sup>Analogously to conventional gradient-search methods, the convergence of the proposed algorithm can be guaranteed by choosing a sufficiently small value of  $\mu$ . Obviously, this parameter also has a direct impact on the speed of convergence, which increases with  $\mu$ .

Select  $\mu$  and  $\alpha$ ; initialize  $\mathbf{w}_T$  and  $\mathbf{w}_R$  (MaxSNR approximation).  
**repeat**  
     **Update of the transmit beamformer**  
     Obtain the equivalent MISO channels  $\mathbf{h}_{\text{MISO}_k}$  with (11).  
     Update  $h_k$  and  $\text{MSE}_k$  for  $k = 1, \dots, N_c$ .  
     Obtain the matrix  $\mathbf{R}_{\text{MISO}_\alpha}$  with (9).  
     Update the beamformer  $\mathbf{w}_T$  with (15).  
     Normalize the solution:  $\mathbf{w}_T = \mathbf{w}_T / \|\mathbf{w}_T\|$ .  
     **Update of the receive beamformer**  
     Obtain the equivalent SIMO channels  $\mathbf{h}_{\text{SIMO}_k}$  with (11).  
     Update  $h_k$  and  $\text{MSE}_k$  for  $k = 1, \dots, N_c$ .  
     Obtain the matrix  $\mathbf{R}_{\text{SIMO}_\alpha}$  with (10).  
     Update the beamformer  $\mathbf{w}_R$  with (16).  
     Normalize the solution:  $\mathbf{w}_R = \mathbf{w}_R / \|\mathbf{w}_R\|$ .  
**until** Convergence

**Algorithm 1:** Proposed beamforming algorithm.

obtaining the solution of the coupled EV problems in (8). The overall technique, which includes a normalization step to force the unit energy constraint on the beamformers, is summarized in Algorithm 1.

Regarding the computational complexity, it is easy to verify that the initialization step has a complexity of order  $\mathcal{O}(n_T^3 n_R^3 + N_c n_T^2 n_R^2)$ , whereas one iteration of the proposed method comes at a cost of approximately  $\mathcal{O}(N_c (n_T + n_R)^2)$ . Thus, 50 iterations of the proposed algorithm in the previous example ( $N_c = 64$  and  $n_T = n_R = 4$ ) would have a cost three orders of magnitude lower than that of the SDR approach previous to the randomization technique.

## V. SIMULATION RESULTS

The performance of the proposed iterative technique is illustrated in this section by means of some Monte Carlo simulations.<sup>4</sup> In all the experiments, we consider a MIMO system with 64 subcarriers and  $n_T = n_R = 4$  transmit and receive antennas. An i.i.d. Rayleigh MIMO channel model with exponential power delay profile has been assumed. In particular, the total power associated to the  $l$ -th tap is

$$E \left[ \|\mathbf{H}[l]\|^2 \right] = (1 - \rho) \rho^l n_T n_R, \quad l = 0, \dots, L_c - 1, \quad (17)$$

where  $L_c$  is the length of the channel impulse response ( $L_c = 16$  in the simulations), and the exponential parameter  $\rho$  has been selected as  $\rho = 0.7$ . We have focused on the MaxSNR ( $\alpha = 0$ ) [1]–[4], MaxCAP ( $\alpha = 1$ ) and MinMSE ( $\alpha = 2$ ) approaches, which have been compared with a SISO system and with a full MIMO scheme applying maximum ratio transmission (MRT) and maximum ratio combining (MRC) per subcarrier (denoted in the figures as Full-MIMO), which can be seen as an upper bound for the performance of any pre-FFT system. Finally, in all the experiments, the step-size has been fixed to  $\mu = 0.1$ , and the convergence criterion is based on the difference between the beamformers in two consecutive iterations. Specifically, the algorithm finishes when the Euclidian distance is lower than  $10^{-3}$ . With these values, the proposed algorithm has never exceeded 50 iterations.

<sup>4</sup>We have verified by means of simulations that the iterative algorithm outperforms the SDR approach with a moderate number of randomizations.

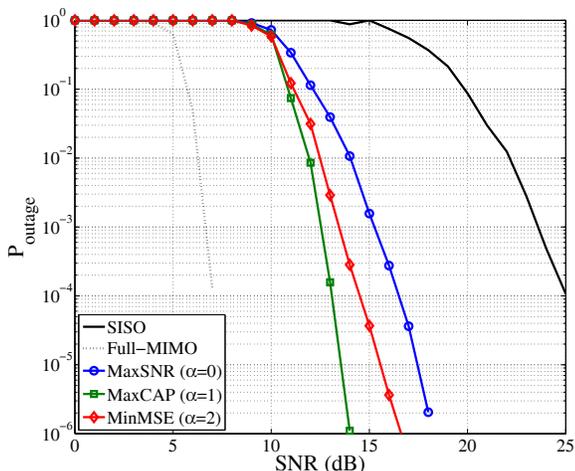


Fig. 3. Outage probability for a transmission rate of 5 bps/Hz.

In the first experiment, we evaluate the outage probability of the system for a transmission rate of 5 bps/Hz, which is shown in Fig. 3. As expected, the best results are provided by the Full-MIMO system, followed by the pre-FFT MaxCAP criterion ( $\alpha = 1$ ), which clearly outperforms the remaining pre-FFT schemes and the SISO system.

The advantage of the MaxCAP and MinMSE criteria over the MaxSNR approach becomes clearer when the system performance is evaluated in terms of bit error rate (BER). In particular, we have adopted the 802.11a standard, which uses  $N_c = 48$  out of 64 subcarriers for data transmission. The information bits are encoded with a convolutional code and block interleaved as specified in the standard. Finally, the receiver is based on a soft Viterbi decoder, and we have selected a transmission rate of 12 Mbps, which implies QPSK signaling and a convolutional code of rate 1/2. Here, the introduction of a channel encoder could induce us to think that the MaxCAP criterion will outperform the remaining approaches. However, as can be seen in Fig. 4, the best results are provided by the MinMSE beamformers. This is due to the fact that we are not using an ideal channel encoder (note that the channel encoder operates on an OFDM symbol basis), which implies that a slight degradation in the capacity can be acceptable in order to obtain a less frequency selective equivalent channel.

## VI. CONCLUSION

In this paper we have proposed a general MIMO beamforming criterion for pre-FFT processing systems. It depends on a single parameter,  $\alpha$ , which establishes a tradeoff between the energy and spectral flatness of the equivalent SISO channel. Several interesting design criteria such as the MaxSNR, the MaxCAP and the MinMSE are just particular cases of the proposed criterion for different values of  $\alpha$ . In general, the proposed criterion results in a non-convex optimization problem, but approximated solutions can be obtained by means of SDR techniques. Furthermore, in order to avoid the high

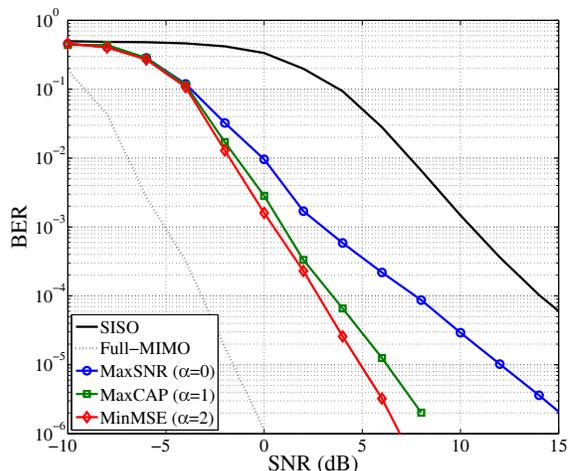


Fig. 4. BER for a 802.11a based system with transmission rate of 12 Mbps. QPSK signaling and convolutional encoder of rate 1/2.

computational cost of SDR techniques, we have proposed a simple and efficient algorithm which, with a proper initialization, provides very good results in practical OFDM-based WLAN standards such as 802.11a. Finally, we can conclude that, in general, it is advisable to increase the spectral flatness of the equivalent SISO channel, even at the expense of a slight degradation of the overall SNR. Future research lines include the extension of the presented criteria to multiuser scenarios.

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