A General Criterion for Analog Tx-Rx Beamforming under OFDM Transmissions

Javier Vía, *Member, IEEE*, Ignacio Santamaría, *Senior Member, IEEE*,
Victor Elvira, *Student Member, IEEE* and Ralf Eickhoff, *Member, IEEE*

***Abstract*—In this paper, we study beamforming schemes for a novel MIMO transceiver, which performs adaptive signal combining in the radio-frequency (RF) domain. Assuming perfect channel knowledge at the receiver side, we consider the problem of designing the transmit and receive RF beamformers under orthogonal frequency division multiplexing (OFDM) transmissions. In particular, a general beamforming criterion is proposed, which depends on a single parameter α. This parameter establishes a tradeoff between the energy of the equivalent SISO channel (after Tx-Rx beamforming) and its spectral flatness. The proposed cost function embraces most reasonable criteria for designing analog Tx-Rx beamformers. Hence, for particular values of α the proposed criterion reduces to the minimization of the mean square error (MSE), the maximization of the system capacity, or the maximization of the received signal-to-noise ratio (SNR). In general, the proposed criterion results in a non-convex optimization problem. However, we show that the problem can be rewritten as a convex cost function subject to a couple of rank-one constraints, and hence it can be approximately solved by semidefinite relaxation (SDR) techniques. Since the computational cost of SDR for this problem is rather high, and building on the observation that the minima of the original problem must be solutions of a pair of coupled eigenvalue problems, we propose yet another simple and efficient gradient search algorithm which, in practice, provides satisfactory solutions with a moderate computational cost. Finally, several numerical examples show the good performance of the proposed technique for both uncoded and 802.11a coded transmissions.**

***Index Terms*—Analog combining, pre-FFT beamforming, orthogonal frequency division multiplexing (OFDM), multiple-input multiple-output (MIMO).**

I. INTRODUCTION

To exploit the benefits (e.g., diversity or multiplexing gain) of multiple-input multiple-output (MIMO) wireless communication systems, all antenna paths must be independently acquired and processed at baseband. Consequently, the hardware costs, size and power consumption of conventional MIMO systems are increased accordingly. These high costs explain in part the delay in the commercial deployment of multiple-antenna wireless transceivers, mainly in handsets or small cost terminals.

A RF-MIMO receiver architecture, shown in Fig. 1, solves some of these problems by shifting spatial signal processing from the base band to the radio-frequency (RF) front-end. The RF-MIMO transmitter operates analogously. The basic idea consists of applying the complex weights $w[n]$ (gain factor and phase shift) to the received signals as shown in Fig. 1. In this way, after combining the weighted RF signals a single stream of data must be acquired and processed and thus the hardware cost and the power consumption are significantly reduced [1]. Although the multiplexing gain of the RF-MIMO transceiver is limited to one (since we transmit/receive a single data stream), in [2] we have shown that other benefits of the MIMO channel such as full spatial diversity or full array gain can be retained by the proposed architecture if proper processing is carried out. Finally, we must point out that, although the details on the practical implementation of the overall system are beyond the scope of this paper, the development of this type of analogue weighting RF circuits in BiCMOS technology suitable for mass fabrication is currently being pursued within the EU funded project MIMAX (*MIMO Systems for Maximum Reliability and Performance*) [3]. In particular, implementation problems such as RF impairments and quantization of the RF weights will translate into a small performance degradation in comparison to other alternative systems, such as pre-FFT processing schemes [4]–[7]. However, these limitations are justified by the significant reduction in hardware cost and power consumption of the analog combining architecture.

From a signal processing point of view, the adaptive antenna combining architecture in Fig. 1 poses several challenging design problems. Specifically, in the case of OFDM transmissions, a conventional MIMO-OFDM receiver can compute the fast Fourier transform (FFT) of each baseband signal, and hence it can apply the optimal processing independently for each subcarrier. However, the new RF-MIMO transceiver uses the same pair of RF weights (or beamformers) for all the subcarriers and therefore the problem is inherently coupled.

In this paper we address the problem of selecting the beamformers under OFDM transmissions with perfect channel state information at the receiver side. In particular, assuming that the channel remains fixed during a sufficiently long time period, the receiver selects the optimal transmit and receive beamformers, and provides the former to the transmitter by means of an ideal feedback channel.

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J. Vía, I. Santamaría and V. Elvira are with the Department of Communications Engineering, University of Cantabria, 39005 Santander, Cantabria, Spain. e-mail: {jvia,nacho,victorea}@gtas.dicom.unican.es. Telephone: +34-942201493. Fax: +34-942201488.

Ralf Eickhoff is with the Chair for Circuit Design and Network Theory, Dresden University of Technology, 01062 Dresden, Germany. e-mail: Ralf.Eickhoff@tu-dresden.de.

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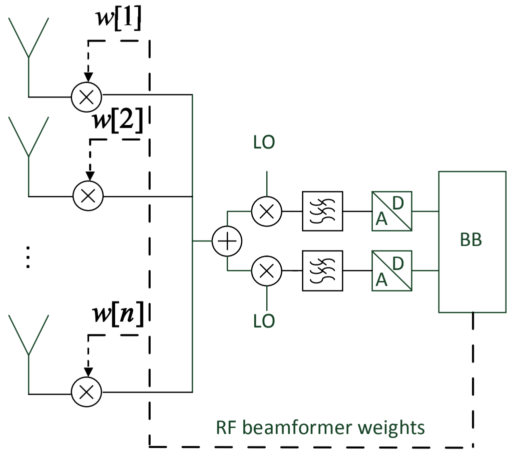

Fig. 1. Analog antenna combining in the RF path for MIMO communications systems. Exemplarily shown for a direct-conversion receiver.

A. Previous Work

In the case of conventional MIMO-OFDM systems, which we also refer to as full MIMO systems or post-FFT processing [6], the joint design of Tx-Rx beamformers has been widely treated in the literature. In particular, in [8], [9] a number of interesting design criteria have been solved in closed-form within the powerful frameworks of convex optimization and majorization [9]–[11].

From the point of view of beamforming design, our analog combining architecture is similar to a pre-FFT scheme, which has been widely studied in the literature [4]–[7]. The main motivation behind pre-FFT schemes is to reduce the cost due to FFT calculations and, to this end, beamforming is shifted before FFT processing. However, most of these pre-FFT schemes only consider receive beamforming, and the design criterion reduces to the maximization of the received SNR (MaxSNR). In this paper we show that other design criteria can provide significantly better performance. Thus, although the present work has been motivated by the novel RF-combining architecture, the obtained results can be directly applied to pre-FFT schemes.

Finally, the statistical eigen-beamforming transmission mode defined in the WiMAX standard [12], [13] is also based on the application of a common transmit beamformer to a set of subcarriers. This idea allows a feedback reduction when compared with a maximum ratio transmission (MRT) approach [12]–[14]. However, analogously to the pre-FFT schemes, the design of the beamformer only considers the MaxSNR criterion.

B. Main Contributions

In this paper we propose a general beamforming criterion which depends on a single parameter α. This parameter establishes a tradeoff between the energy and the spectral flatness of the equivalent SISO channel (after Tx-Rx beamforming). Furthermore, it allows us to obtain several interesting beamforming criteria, such as the maximization of the received SNR (MaxSNR, $\alpha = 0$) [4]–[7], the maximization of the system capacity (MaxCAP, $\alpha = 1$) [15], and the minimization of the mean square error (MSE) associated to the optimal linear receiver (MinMSE, $\alpha = 2$) [16]. Interestingly, although all the criteria reduce to the MaxSNR approach in the low SNR regime, they significantly differ for moderate or high SNRs. In particular, as α increases, the proposed criterion sacrifices part of the received SNR in order to improve the response of the worst data carriers, which translates into significant advantages in terms of capacity, MSE, and bit error rate (BER).

Analogously to the MaxSNR approach for pre-FFT MIMO schemes [5], the proposed beamforming criterion results in a non-convex optimization problem and has no closed-form solution. In this paper, we analyze the associated non-convex optimization problem and show that, in certain cases, it can be approximately solved by means of semidefinite relaxation (SDR) techniques. Furthermore, in order to avoid the high complexity cost associated to SDR techniques, we propose a suboptimal gradient search algorithm which, in combination with a very effective initialization technique, provides very accurate results in most of the practical cases. Finally, several simulation examples show the advantage of the proposed technique over the MaxSNR approach for both coded and uncoded transmissions.

C. Organization

The data model and problem statement are presented in Section II. In Section III we present the general beamforming criterion and discuss its main properties. The associated optimization problem is analyzed in Section IV, where we also introduce the proposed beamforming algorithm. In Section V, the good performance of the proposed method is illustrated by means of several simulation examples. Finally, the main conclusions are summarized in Section VI, whereas some technical details have been relegated to the appendix.

II. Preliminaries

A. Notation

Throughout this paper we will use bold-faced upper case letters to denote matrices, bold-faced lower case letters for column vector, and light-faced lower case letters for scalar quantities. Superscripts $(\cdot)^T$, $(\cdot)^H$ and $(\cdot)^*$ denote transpose, Hermitian and complex conjugate, respectively. $\|\mathbf{A}\|$, $\mathrm{Tr}(\mathbf{A})$, $\mathrm{rank}(\mathbf{A})$ and $\mathrm{vec}(\mathbf{A})$ will denote, respectively, the Frobenius norm, trace, rank, and column-wise vectorized version of matrix $\mathbf{A}$. $\mathbf{A} \succeq \mathbf{0}$ means that $\mathbf{A}$ is Hermitian and positive semidefinite, whereas $\mathbf{v}_{\max}(\mathbf{A})$ is the principal eigenvector of the Hermitian positive semidefinite matrix $\mathbf{A}$. Finally, $\mathbf{I}$ and $\mathbf{0}$ are the identity and zero matrices of the required dimensions, and $E[\cdot]$ denotes the expectation operator.

B. Main Assumptions

The main assumptions in this paper are the following:

- We do not consider RF impairments such as I/Q imbalance, imperfections in the RF circuitry, or quantization errors in the RF weights. However, we must point out that recent advances in RF integrated circuits designed in SiGe-BiCMOS technology [17] have made feasible the combination of RF signals using precise phase shifters with 360° control range and an amplitude dynamic range of more than 20 dB. Therefore, as it will be shown in the simulations section, we should not expect a high impact of RF impairments in the performance of the proposed architecture.

- As shown in Fig. 1, we only consider one RF chain (equivalently, one FFT in the pre-FFT processing scenario) at the transmitter and receiver. The extension of the results in this paper to the case of multiple data streams using multiple analog beamformers in parallel, each one followed by the corresponding RF chain and FFT block, would follow the lines in [18], [19] for the pre-FFT MaxSNR case, and will be considered in the future.
- The MIMO channel and noise variance are perfectly known at the receiver side. We do not consider channel estimation errors due to the noise, the limited number of pilots, or the channel estimation process. On the other hand, note that the channel estimation process can be reduced to the sequential estimation of several single-input single-output (SISO) frequency selective channels. Additionally, in the case of pre-FFT systems and for the MaxSNR criterion, the optimal beamformers can be extracted from the signal covariance matrix, i.e., the knowledge of the channel and noise variance is not required [5], [7].
- The channel is unknown at the transmitter. Analogously to the WiMAX statistical eigen-beamforming transmission mode [12], [13], the only feedback from the receiver is the optimal transmit beamformer. Therefore, the feedback is significantly reduced in comparison with the transmission of all the coefficients of the frequency-selective MIMO channel. On the other hand, we must note that under this assumption, the transmitter cannot apply adaptive power loading techniques. The design of the beamformers under channel knowledge at the transmitter side with adaptive power loading, constitutes also an interesting topic for future research.

C. System Model

Let us consider a RF-MIMO system with n_T transmit and n_R receive antennas, and with unit-energy transmit and receive beamformers defined by the RF weights in Fig. 1. Assuming a transmission scheme based on OFDM with N_c data-carriers and using a cyclic prefix longer than the channel impulse response, the communication system after Tx-Rx radio frequency beamforming may be decomposed into the following set of parallel and non-interfering single-input single-output (SISO) equivalent channels

$$y_k = h_k s_k + n_k, \qquad k = 1, \ldots, N_c,$$

where $y_k \in \mathbb{C}$ is the observation associated to the k-th data carrier, n_k represents the complex circular i.i.d. Gaussian noise with zero mean and variance σ^2, s_k is the transmitted signal, and h_k is the equivalent channel after Tx-Rx beamforming, which is given by

$$h_k = \mathbf{w}_R^H \mathbf{H}_k \mathbf{w}_T, \qquad k = 1, \ldots, N_c,$$

where $\mathbf{w}_T \in \mathbb{C}^{n_T \times 1}$ and $\mathbf{w}_R \in \mathbb{C}^{n_R \times 1}$ are the transmit and receive beamformers, and $\mathbf{H}_k \in \mathbb{C}^{n_R \times n_T}$ represents the MIMO channel for the k-th data-carrier.

D. LMMSE Receiver

Although the results in this paper are not restricted to a particular receiver, it will be useful to review the linear minimum mean square error (LMMSE) receiver. In particular, under perfect knowledge of the equivalent channel, and assuming unit transmit power per data carrier ($E[|s_k|^2] = 1$), the MMSE estimate of s_k is

$$\hat{s}_k = \frac{h_k^* y_k}{|h_k|^2 + \sigma^2},$$

which yields a per-carrier MSE

$$\text{MSE}_k = E\left[\left| \hat{s}_k - s_k \right|^2 \right] = \frac{1}{1 + \gamma |h_k|^2}, \qquad k = 1, \ldots, N_c,$$

where $\gamma = 1/\sigma^2$ is defined as the (expected) signal to noise ratio (SNR) at the transmitter side.

E. Problem Statement

Conventional MIMO-OFDM baseband schemes have access to the signals at each one of the transmitting/receiving antennas and, consequently, can obtain a different pair of beamformers for each subcarrier. However, with the novel analog RF combining architecture a per-carrier beamforming design is not possible since all the orthogonal MIMO channels $\mathbf{H}_k$ are affected by the same pair of beamformers. Notice that with the RF combining architecture a single FFT must be computed after the analog beamforming (at the receiver side), which notably simplifies the hardware and the system computational complexity, but also complicates the beamforming design problem due to the coupling among subcarriers. This coupling imposes some tradeoffs and represents the main challenge for the design of the beamformers. In the following section, we tackle the problem of joint Tx-Rx analog beamforming design using a unifying cost function which, by changing a single parameter, encompasses several interesting design criteria.

III. General Analog Beamforming Criterion

In this section we introduce a general criterion for the design of the Tx-Rx beamformers under perfect knowledge of the MIMO channel $\mathbf{H}_k$, as well as the noise variance, at the receiver side. Specifically, we propose to minimize the following cost function

$$f_\alpha(\mathbf{w}_T, \mathbf{w}_R) = \frac{1}{\alpha - 1} \log \left(\frac{1}{N_c} \sum_{k=1}^{N_c} \text{MSE}_k^{\alpha - 1} \right), \qquad (1)$$

where α is a real parameter which controls the overall system performance. Thus, our optimization problem can be written as

$$\begin{aligned} \arg\min_{\mathbf{w}_T,\mathbf{w}_R} \quad & f_\alpha(\mathbf{w}_T,\mathbf{w}_R) \\ \text{subject to} \quad & \|\mathbf{w}_T\| = 1, \\ & \|\mathbf{w}_R\| = 1. \end{aligned} \qquad (2)$$

It is interesting to mention that (1) structurally resembles the definition of Renyi's entropy of order α for a discrete random variable [20]. This is a parametric family of entropy measures that include conventional Shannon's entropy definition as a limiting case when α tends to 1, and which has recently been used by Principe and co-workers in a number of applications such as blind source separation and blind deconvolution/equalization [21], [22].

Before addressing the optimization problem, let us analyze some interesting choices of α, which will help us to shed some light into the properties of the cost function (1).

A. Particular Cases

1) MaxSNR ($\alpha = 0$): If the parameter α is set to zero, the optimization problem in (2) can be rewritten as

$$\arg\max_{\mathbf{w}_T,\mathbf{w}_R} \quad \frac{1}{N_c}\sum_{k=1}^{N_c} |h_k|^2 \quad \text{s. t.} \quad \|\mathbf{w}_T\| = \|\mathbf{w}_R\| = 1,$$

i.e., the proposed criterion reduces to the maximization of the energy of the equivalent channel or, in other words, to the maximization of the received SNR. This problem has been previously addressed by other authors in the contexts of analog combining [23] and pre-FFT schemes [4]–[7], and it is also closely related to the statistical eigen beamforming transmission mode defined in the WiMAX standard [12], [13].

2) MaxCAP ($\alpha = 1$): When α approaches 1, it can be easily shown by direct application of the L'Hopital's rule, that the proposed criterion reduces to

$$\arg\max_{\mathbf{w}_T,\mathbf{w}_R} \quad \frac{1}{N_c}\sum_{k=1}^{N_c} \log\left(1+\gamma|h_k|^2\right) \quad \text{s. t.} \quad \|\mathbf{w}_T\| = \|\mathbf{w}_R\| = 1,$$

which represents the capacity of the equivalent SISO channel after beamforming.

3) MinMSE ($\alpha = 2$): In this case, (2) is equivalent to

$$\arg\min_{\mathbf{w}_T,\mathbf{w}_R} \quad \frac{1}{N_c}\sum_{k=1}^{N_c} \mathrm{MSE}_k \quad \text{s. t.} \quad \|\mathbf{w}_T\| = \|\mathbf{w}_R\| = 1,$$

i.e., the proposed criterion amounts to minimizing the overall MSE of the optimal linear receiver. Moreover, it can be proved that, in the important case of quadrature amplitude modulation (QAM) constellations, and under optimal linear precoding of the information symbols, the minimization of the MSE is equivalent to the minimization of the bit error rate (BER) [9], [24].

Although in this paper we mainly focus on the three previous values of α, it should be noted that any other choice would be in principle possible. In particular, two other interesting cases are the following:

4) MaxMin ($\alpha = \infty$): In this case, the summation in (1) is dominated by the worst data-carrier, i.e., by that with the smallest $|h_k|^2$. Therefore, for $\alpha \to \infty$, the proposed criterion reduces to the optimization of the worst data-carrier. Interestingly, in the particular single-input multiple-output (SIMO) and multiple-input single-output (MISO) cases, the proposed criterion is mathematically identical to that of the MaxMin fair multicast beamforming problem, which has been proven to be NP-hard [25], [26].

5) MaxMax ($\alpha = -\infty$): For $\alpha < 1$ the proposed criterion can be rewritten as

$$\arg\max_{\mathbf{w}_T,\mathbf{w}_R} \quad \sum_{k=1}^{N_c} \left(1+\gamma|h_k|^2\right)^{1-\alpha} \quad \text{s. t.} \quad \|\mathbf{w}_T\| = \|\mathbf{w}_R\| = 1,$$

then, it is easy to see that, when $\alpha \to -\infty$, the summation is dominated by the largest $|h_k|$. Therefore, the proposed criterion reduces to the optimization of the best data-carrier.[1] Interestingly, in this case the optimal beamformers can be obtained in closed-form as the left and right singular vectors associated to the largest eigenvalue of all the MIMO channels $\mathbf{H}_k$ $(k = 1, \ldots, N_c)$.

B. Main Properties

In this subsection, the main properties of the proposed beamforming criterion are summarized. Let us start by analyzing the performance of the proposed method in the low SNR regime.

Property 1: In the low SNR regime ($\gamma \to 0$), the proposed criterion reduces to the MaxSNR approach regardless of α.

Proof: The proof is based on the first order Taylor series approximation of $f_\alpha(\mathbf{w}_T,\mathbf{w}_R)$ with respect to γ

$$f_\alpha(\mathbf{w}_T,\mathbf{w}_R) \simeq -\gamma \sum_{k=1}^{N_c} |h_k|^2.$$

Thus, the minimization of $f_\alpha(\mathbf{w}_T,\mathbf{w}_R)$ reduces to the maximization of the equivalent channel energy or received SNR. ■

Obviously, the different criteria significantly differ for moderate or high SNRs. The following property ensures the Pareto optimality (or *efficiency*) [10] of the obtained solutions. Specifically, a feasible pair of beamformers ($\tilde{\mathbf{w}}_T$, $\tilde{\mathbf{w}}_R$) is said to be Pareto optimal with respect to the individual channel energies ($|h_k|^2$) or MSEs (MSE_k) iff there does not exist another feasible pair ($\mathbf{w}_T$, $\mathbf{w}_R$) satisfying

$$|\mathbf{w}_R^H \mathbf{H}_k \mathbf{w}_T|^2 \geq |\tilde{\mathbf{w}}_R^H \mathbf{H}_k \tilde{\mathbf{w}}_T|^2, \qquad k = 1, \ldots, N_c, \qquad (3)$$

with at least one strict inequality.

Property 2: The solutions ($\tilde{\mathbf{w}}_T$, $\tilde{\mathbf{w}}_R$) of the proposed beamforming criterion are Pareto optimal points of the vector optimization problem based on the individual channel energies ($|h_k|^2$) or MSEs (MSE_k).

Proof: The proof follows directly from the fact that $f_\alpha(\mathbf{w}_T,\mathbf{w}_R)$ decreases with $|h_k|^2$ $(k = 1, \ldots, N_c)$. Thus, if

[1] This would be the optimal criterion for an adaptive power loading scheme with only one active subcarrier.

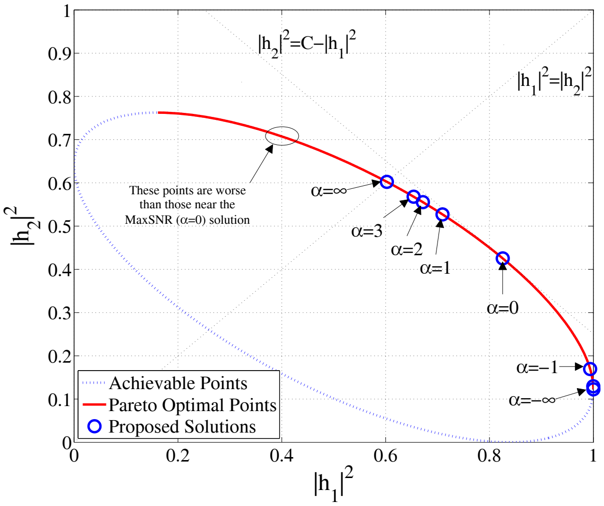

Fig. 2. Toy example with a 1×2 SIMO system with $N_c = 2$ subcarriers. The figure shows the set of achievable points with $\|\mathbf{w}_R\| = 1$ (dotted line), the Pareto optimal points (solid line), and the solutions associated to the proposed criterion (the section of the curve marked with circles).

a feasible point $\mathbf{w}_T$, $\mathbf{w}_R$ satisfies the inequalities in (3), then $f_\alpha(\mathbf{w}_T, \mathbf{w}_R) < f_\alpha(\tilde{\mathbf{w}}_T, \tilde{\mathbf{w}}_R)$. ■

Here we must note that the converse of Property 2 is not true in general, i.e., not all the Pareto optimal points (with respect to the N_c channel energies $|h_k|^2$ or MSEs) are solutions of the proposed criterion for some α. This fact is illustrated by means of a toy example consisting of a SIMO system with $N_c = 2$ data carriers, $n_R = 2$ receive antennas, real beamformers and subcarrier channels given by

$$\mathbf{H}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \qquad \mathbf{H}_2 = \begin{bmatrix} 0.35 \\ -0.8 \end{bmatrix}.$$

Fig. 2 shows the set of achievable points ($|h_1|^2$,$|h_2|^2$) with unit energy beamformers $\mathbf{w}_R$, where we can see that the solutions of the proposed criterion for different values of α are only a subset of the Pareto optimal solutions. However, it should be pointed out that those points in the Pareto boundary that are not achievable by our criterion might not be of practical interest. As an example, consider the Pareto points in Fig. 2 near $|h_1|^2 = 0.4$, $|h_2|^2 = 0.7$. Clearly, these points are worse solutions than those near $|h_1|^2 = 0.8$, $|h_2|^2 = 0.4$, which are solutions of the proposed criterion for some $\alpha \simeq 0$.

The above observation is a direct consequence of the fact that the ordering of the energies is irrelevant for the performance of the equivalent channel. A more sensible comparison between feasible pairs of beamformers can be established with the help of some standard majorization results [9], [11].

Property 3: Let us define $P_\beta = \sum_{k=1}^{N_c} \text{MSE}_k^{\beta-1}$ and the vector $\mathbf{p}_\beta(\mathbf{w}_T, \mathbf{w}_R) = [p_{\beta,1}, \ldots, p_{\beta,N_c}]$ with elements[2]

$$p_{\beta,k} = \frac{\text{MSE}_k^{\beta-1}}{P_\beta}, \qquad k = 1, \ldots, N_c.$$

[2]Note that, since $0 \leq p_{\beta,k} \leq 1$ and $\sum_{k=1}^{N_c} p_{\beta,k} = 1$, $\mathbf{p}_\beta(\mathbf{w}_T, \mathbf{w}_R)$ can be seen as the probability mass function of a discrete random variable.

Then, the cost function $f_\alpha(\mathbf{w}_T, \mathbf{w}_R)$ can be rewritten as

$$f_\alpha(\mathbf{w}_T, \mathbf{w}_R) = f_\beta(\mathbf{w}_T, \mathbf{w}_R) + g_{\alpha,\beta}\left(\mathbf{p}_\beta(\mathbf{w}_T, \mathbf{w}_R)\right), \quad (4)$$

where $g_{\alpha,\beta}(\mathbf{p}_\beta(\mathbf{w}_T, \mathbf{w}_R))$ is a function satisfying the following properties:

1) $g_{\alpha,\beta}(\mathbf{p}_\beta(\mathbf{w}_T, \mathbf{w}_R)) = -g_{\beta,\alpha}(\mathbf{p}_\alpha(\mathbf{w}_T, \mathbf{w}_R))$.
2) $g_{\alpha,\beta}(\mathbf{p}_\beta(\mathbf{w}_T, \mathbf{w}_R))$ increases with α.
3) For $\alpha > \beta$: $g_{\alpha,\beta}(\mathbf{p}_\beta(\mathbf{w}_T, \mathbf{w}_R))$ is a Schur-convex function [9], [11], which attains its minimum $g_{\alpha,\beta}(\mathbf{p}_\beta(\mathbf{w}_T, \mathbf{w}_R)) = 0$ iff $p_{\beta,k} = 1/N_c$ ($\forall k$).

Proof: See the appendix. ■

Property 3 allows us to shed some light into the effect of the parameter α. Firstly, we must note that P_β is a global performance measure directly related to the cost function

$$f_\beta(\mathbf{w}_T, \mathbf{w}_R) = \frac{1}{\beta - 1} \log\left(\frac{P_\beta}{N_c}\right),$$

whereas $\mathbf{p}_\beta$ represents the distribution of P_β along the data carriers.

From eq. (4), we observe that $f_\alpha(\mathbf{w}_T, \mathbf{w}_R)$ can be seen as a penalized version of the cost function $f_\beta(\mathbf{w}_T, \mathbf{w}_R)$. Furthermore, taking into account the Schur-convexity of the penalty term for $\alpha > \beta$, we can conclude that $g_{\alpha,\beta}(\mathbf{p}_\beta(\mathbf{w}_T, \mathbf{w}_R))$ penalizes the *spreading* of the elements of $\mathbf{p}_\beta(\mathbf{w}_T, \mathbf{w}_R)$, i.e., it can be interpreted as an alternative measure of the spectral flatness of the equivalent channel [9], [11], [27]. On the other hand, since the penalty term $g_{\alpha,\beta}(\mathbf{p}_\beta(\mathbf{w}_T, \mathbf{w}_R))$ increases with α, we can say that, when α increases, the proposed beamforming criterion tends to flatten the equivalent channel, i.e., the critical data carriers (those with the smallest $|h_k|$) are improved at the expense of a slight degradation of $f_\beta(\mathbf{w}_T, \mathbf{w}_R)$. Finally, taking into account that $g_{\alpha,\beta}(\mathbf{p}_\beta(\mathbf{w}_T, \mathbf{w}_R)) = -g_{\beta,\alpha}(\mathbf{p}_\alpha(\mathbf{w}_T, \mathbf{w}_R))$, we can obtain similar conclusions for the case $\alpha < \beta$. In particular, as α decreases, the proposed criterion allows a small increase in $f_\beta(\mathbf{w}_T, \mathbf{w}_R)$ in order to obtain a higher spread of the terms in $\mathbf{p}_\beta(\mathbf{w}_T, \mathbf{w}_R)$.

As an example, consider the MaxSNR ($\alpha = 0$), MaxCAP ($\alpha = 1$) and MinMSE ($\alpha = 2$) cases. Thus, when the parameter increases from $\alpha = 1$ to $\alpha = 2$, the worst subcarriers are improved at the expense of a slight decrease of the equivalent channel capacity. On the other hand, when the design parameter decreases from $\alpha = 1$ to $\alpha = 0$, the energy of the equivalent channel increases at the expense of a reduction in the spectral flatness as well as in capacity.

IV. Optimization Problem and Proposed Algorithm

In this section, the optimization problem derived from the proposed beamforming criterion is analyzed. Although the optimization problem is in general non-convex, approximate solutions can be obtained by means of semidefinite relaxation (SDR) techniques for $\alpha \geq 0$. However, the computational cost associated to SDR techniques can be very high even for a moderate number of data subcarriers and antennas. For this reason, we propose a simple gradient search method which is initialized using a closed-form approximation of the MaxSNR solution. As it will be shown in Section V, this method provides very accurate results.

A. Optimization Problem

Let us start by rewriting the optimization problem in (2) as

$$\arg\min_{\mathbf{W}} \quad \frac{1}{\alpha-1}\log\left(\frac{1}{N_c}\sum_{k=1}^{N_c}\left(1+\gamma\left|\mathrm{Tr}(\mathbf{H}_k\mathbf{W})\right|^2\right)^{1-\alpha}\right), \tag{5}$$

$$\text{subject to} \quad \|\mathbf{W}\| = 1, \quad \mathrm{rank}(\mathbf{W}) = 1,$$

where $\mathbf{W} = \mathbf{w}_T\mathbf{w}_R^H$ is the rank-one Tx-Rx beamforming matrix.[3] Although the solution of the above problem can be obtained in closed-form in some particular cases (see Table I), in general this is a very difficult problem due to the two following reasons. Firstly, the rank-one constraint on the beamforming matrix $\mathbf{W}$ is not convex. Secondly, although the relaxation of the rank-one constraint will allow us to obtain good initialization points for the beamvectors, in general the cost function remains still non-convex for most values of α,[4] which precludes the application of standard convex optimization techniques [10].

B. Analysis of the Cost Function Minima

Although the non-convexity of the optimization problem precludes obtaining a closed-form solution, we can gain some insight by applying the Lagrange multipliers method and thus finding conditions that must be satisfied by any local minima. In this subsection, we show that the local minima of our optimization problem are closely related to that of a weighted energy maximization problem. This relationship can be easily established by combining the two following lemmas.

Lemma 1: The local minima of the optimization problem in (2) are solutions of the following coupled eigenvalue (EV) problems

$$\mathbf{R}_{\mathrm{MISO}_\alpha}\mathbf{w}_T = \lambda\mathbf{w}_T, \qquad \mathbf{R}_{\mathrm{SIMO}_\alpha}\mathbf{w}_R = \lambda\mathbf{w}_R, \tag{6}$$

where $\lambda = \sum_{k=1}^{N_c}\mathrm{MSE}_k^\alpha|h_k|^2$,

$$\mathbf{R}_{\mathrm{MISO}_\alpha} = \sum_{k=1}^{N_c}\mathrm{MSE}_k^\alpha\mathbf{h}_{\mathrm{MISO}_k}\mathbf{h}_{\mathrm{MISO}_k}^H, \tag{7}$$

$$\mathbf{R}_{\mathrm{SIMO}_\alpha} = \sum_{k=1}^{N_c}\mathrm{MSE}_k^\alpha\mathbf{h}_{\mathrm{SIMO}_k}\mathbf{h}_{\mathrm{SIMO}_k}^H, \tag{8}$$

can be seen as weighted covariance matrices and

$$\mathbf{h}_{\mathrm{MISO}_k} = \mathbf{H}_k^H\mathbf{w}_R, \qquad \mathbf{h}_{\mathrm{SIMO}_k} = \mathbf{H}_k\mathbf{w}_T, \tag{9}$$

are the MISO (SIMO) channels after fixing the receive (transmit) beamformer.

Proof: Let us write the Lagrangian of (2) as

$$\mathcal{L}(\mathbf{w}_T,\mathbf{w}_R,\lambda_T,\lambda_R) = f_\alpha(\mathbf{w}_T,\mathbf{w}_R) + \lambda_T\left(\|\mathbf{w}_T\|^2-1\right) + \lambda_R\left(\|\mathbf{w}_R\|^2-1\right),$$

where λ_T and λ_R are the Lagrange multipliers. Solving with respect to $\mathbf{w}_T$ and $\mathbf{w}_R$ we obtain

$$\nabla_{\mathbf{w}_T^*}f_\alpha(\mathbf{w}_T,\mathbf{w}_R) = -\lambda_T\mathbf{w}_T, \tag{10}$$

$$\nabla_{\mathbf{w}_R^*}f_\alpha(\mathbf{w}_T,\mathbf{w}_R) = -\lambda_R\mathbf{w}_R, \tag{11}$$

where the gradient of the cost function $f_\alpha(\mathbf{w}_T,\mathbf{w}_R)$ with respect to the transmit and receive beamformers is given by

$$\nabla_{\mathbf{w}_T^*}f_\alpha(\mathbf{w}_T,\mathbf{w}_R) = -\frac{\gamma}{\sum_{k=1}^{N_c}\mathrm{MSE}_k^{\alpha-1}}\mathbf{R}_{\mathrm{MISO}_\alpha}\mathbf{w}_T,$$

$$\nabla_{\mathbf{w}_R^*}f_\alpha(\mathbf{w}_T,\mathbf{w}_R) = -\frac{\gamma}{\sum_{k=1}^{N_c}\mathrm{MSE}_k^{\alpha-1}}\mathbf{R}_{\mathrm{SIMO}_\alpha}\mathbf{w}_R.$$

Now, left-multiplying (10) and (11) by $\mathbf{w}_T^H$ and $\mathbf{w}_R^H$, and taking into account the unit-energy constraint on the beamformers, we obtain

$$\lambda_T = \gamma\frac{\mathbf{w}_T^H\mathbf{R}_{\mathrm{MISO}_\alpha}\mathbf{w}_T}{\sum_{k=1}^{N_c}\mathrm{MSE}_k^{\alpha-1}}, \qquad \lambda_R = \gamma\frac{\mathbf{w}_R^H\mathbf{R}_{\mathrm{SIMO}_\alpha}\mathbf{w}_R}{\sum_{k=1}^{N_c}\mathrm{MSE}_k^{\alpha-1}},$$

which combined with (10) and (11) yields

$$\mathbf{R}_{\mathrm{MISO}_\alpha}\mathbf{w}_T = \left(\mathbf{w}_T^H\mathbf{R}_{\mathrm{MISO}_\alpha}\mathbf{w}_T\right)\mathbf{w}_T,$$

$$\mathbf{R}_{\mathrm{SIMO}_\alpha}\mathbf{w}_R = \left(\mathbf{w}_R^H\mathbf{R}_{\mathrm{SIMO}_\alpha}\mathbf{w}_R\right)\mathbf{w}_R.$$

Finally, from (7) and (8) it is easy to see that

$$\mathbf{w}_T^H\mathbf{R}_{\mathrm{MISO}_\alpha}\mathbf{w}_T = \mathbf{w}_R^H\mathbf{R}_{\mathrm{SIMO}_\alpha}\mathbf{w}_R = \sum_{k=1}^{N_c}\mathrm{MSE}_k^\alpha|h_k|^2 = \lambda,$$

which implies $\lambda_T = \lambda_R$ and proves (6). ∎

Lemma 2: Consider the following weighted energy maximization problem[5]

$$\arg\min_{\mathbf{w}_T,\mathbf{w}_R} \quad -\frac{1}{N_c}\sum_{k=1}^{N_c}c_k|h_k|^2 \quad \text{s. t.} \quad \|\mathbf{w}_T\| = \|\mathbf{w}_R\| = 1, \tag{12}$$

with $\mathbf{c} = [c_1,\ldots,c_{N_c}]^T \in \mathbb{R}^{N_c\times 1}$. The local minima of (12) are also solutions of the coupled EV problems

$$\mathbf{R}_{\mathrm{MISO}_\mathbf{c}}\mathbf{w}_T = \lambda\mathbf{w}_T, \qquad \mathbf{R}_{\mathrm{SIMO}_\mathbf{c}}\mathbf{w}_R = \lambda\mathbf{w}_R,$$

where $\lambda = \sum_{k=1}^{N_c}c_k|h_k|^2$ and

$$\mathbf{R}_{\mathrm{MISO}_\mathbf{c}} = \sum_{k=1}^{N_c}c_k\mathbf{h}_{\mathrm{MISO}_k}\mathbf{h}_{\mathrm{MISO}_k}^H,$$

$$\mathbf{R}_{\mathrm{SIMO}_\mathbf{c}} = \sum_{k=1}^{N_c}c_k\mathbf{h}_{\mathrm{SIMO}_k}\mathbf{h}_{\mathrm{SIMO}_k}^H.$$

Proof: The proof is analogous to that of Lemma 1. ∎

Combining the two previous lemmas, we can conclude that the local minima of the proposed optimization problem are also local minima of (12) with weights $c_k = \mathrm{MSE}_k^\alpha$. This corroborates our previous finding about the proposed cost function, i.e., for $\alpha > 0$ the higher weights are given to the subcarriers with a worst response (larger MSE_k). In other words, for $\alpha > 0$ part of the SNR is sacrificed in order to improve the worst data carriers, and the contrary happens for $\alpha < 0$.

[3] Note that, given a solution $\mathbf{W}$ of (5), the transmit and receive beamformers satisfying $\|\mathbf{w}_T\| = \|\mathbf{w}_R\| = 1$ can be easily obtained as the singular vectors of $\mathbf{W}$.

[4] Specifically, the smoothness of the cost function decreases when $|\alpha|$ increases.

[5] Note that in the case of equal weights ($c_k = c, \forall k$) the weighted-energy maximization problem reduces to the MaxSNR criterion.

TABLE I
PARTICULAR CASES WITH CLOSED-FORM SOLUTIONS.

System	$\mathbf{w}_T$	$\mathbf{w}_R$
SIMO with $\alpha = 0$ or $\gamma \simeq 0$	1	MRC
MISO with $\alpha = 0$ or $\gamma \simeq 0$	MRT	1
MIMO flat fading	MRT	MRC

In general, the EV problems in (6) cannot be easily solved due to the fact that the matrices $\mathbf{R}_{\text{MISO}_\alpha}$ and $\mathbf{R}_{\text{SIMO}_\alpha}$ depend on the beamformers. However, in the particular MaxSNR or low-SNR cases[6] ($\alpha = 0$ or $\gamma \simeq 0$, respectively) with MISO or SIMO systems, the optimal solution can be obtained in closed-form. Specifically, the transmit/receive beamformer [4]–[7], [12], [13], [23] is given by the principal eigenvector of the matrices $\mathbf{R}_{\text{MISO}_0}$ and $\mathbf{R}_{\text{SIMO}_0}$ defined in (7) and (8), which resembles the maximum ratio transmission (MRT) or maximum ratio combining (MRC) technique [14]. Finally, as summarized in Table I, in the case of flat channels ($\mathbf{H}_k = \mathbf{H}$, $\forall k$), the optimal beamformers are given by the left and right singular vectors of $\mathbf{H}$, i.e., as expected, the optimal solution reduces to the MRT-MRC MIMO beamforming technique regardless of α.

C. Approximated Solution based on Semidefinite Relaxation

In this subsection we show that an approximated solution to the non-convex optimization problem in (5) can be obtained by applying semidefinite relaxation (SDR) techniques. Let us start by introducing the following lemma, which presents a reformulation of (5) suitable for SDR.

Lemma 3: The optimization problem in (5) is equivalent to

$$\begin{aligned} \underset{\mathbf{W},\tilde{\mathbf{W}},\gamma_1,\ldots,\gamma_{N_c}}{\arg\min} \quad & \frac{1}{\alpha-1}\sum_{k=1}^{N_c}\left(1+\gamma_k\right)^{1-\alpha}, \\ \text{subject to} \quad & \gamma \text{Tr}(\tilde{\mathbf{H}}_k\tilde{\mathbf{W}}) = \gamma_k, \qquad k=1,\ldots,\mathbf{N}_c \\ & \text{Tr}(\tilde{\mathbf{W}}) = 1, \\ & \tilde{\mathbf{W}} \succeq \mathbf{0}, \\ & \text{vec}(\mathbf{W}) = \mathbf{v}_{\max}(\tilde{\mathbf{W}}), \\ & \text{rank}(\tilde{\mathbf{W}}) = 1, \\ & \text{rank}(\mathbf{W}) = 1, \end{aligned} \tag{13}$$

where $\tilde{\mathbf{H}}_k = \mathbf{h}_k\mathbf{h}_k^H$, $\mathbf{h}_k = \text{vec}(\mathbf{H}_k^H)$ can be seen as a virtual SIMO or MISO channel with $n_T n_R$ antennas, $\mathbf{w} = \text{vec}(\mathbf{W})$, and $\tilde{\mathbf{W}} = \mathbf{w}\mathbf{w}^H$ is the associated $n_T n_R \times n_T n_R$ rank-one beamforming matrix. On the other hand, $\mathbf{v}_{\max}(\tilde{\mathbf{W}})$ denotes the eigenvector corresponding to the maximum eigenvalue of $\tilde{\mathbf{W}}$.

[6]Property 1 ensures that in the low SNR regime the proposed criterion reduces to the MaxSNR approach regardless of α.

Proof: Taking into account the monotonicity of the log function, the problem in (5) can be rewritten as

$$\begin{aligned} \underset{\mathbf{W},\mathbf{w}}{\arg\min} \quad & \frac{1}{\alpha-1}\sum_{k=1}^{N_c}\left(1+\gamma\left|\mathbf{h}_k^H\mathbf{w}\right|^2\right)^{1-\alpha}, \\ \text{subject to} \quad & \|\mathbf{w}\| = 1, \\ & \text{vec}(\mathbf{W}) = \mathbf{w}, \\ & \text{rank}(\mathbf{W}) = 1, \end{aligned}$$

or, equivalently,

$$\begin{aligned} \underset{\mathbf{W},\mathbf{w},\tilde{\mathbf{W}}}{\arg\min} \quad & \frac{1}{\alpha-1}\sum_{k=1}^{N_c}\left(1+\gamma\text{Tr}(\tilde{\mathbf{H}}_k\tilde{\mathbf{W}})\right)^{1-\alpha}, \\ \text{subject to} \quad & \text{Tr}(\tilde{\mathbf{W}}) = 1, \\ & \tilde{\mathbf{W}} = \mathbf{w}\mathbf{w}^H, \\ & \text{vec}(\mathbf{W}) = \mathbf{w}, \\ & \text{rank}(\mathbf{W}) = 1. \end{aligned}$$

Finally, introducing the N_c "slack" variables γ_k ($k = 1,\ldots,N_c$) we obtain

$$\begin{aligned} \underset{\mathbf{W},\mathbf{w},\tilde{\mathbf{W}},\gamma_1,\ldots,\gamma_{N_c}}{\arg\min} \quad & \frac{1}{\alpha-1}\sum_{k=1}^{N_c}\left(1+\gamma_k\right)^{1-\alpha}, \\ \text{subject to} \quad & \gamma\text{Tr}(\tilde{\mathbf{H}}_k\tilde{\mathbf{W}}) = \gamma_k, \qquad k=1,\ldots,\mathbf{N}_c \\ & \text{Tr}(\tilde{\mathbf{W}}) = 1, \\ & \tilde{\mathbf{W}} = \mathbf{w}\mathbf{w}^H, \\ & \text{vec}(\mathbf{W}) = \mathbf{w}, \\ & \text{rank}(\mathbf{W}) = 1, \end{aligned}$$

and writing $\mathbf{w}$ as a function of $\mathbf{W}$ and $\tilde{\mathbf{W}}$, the above problem can be rewritten as (13). ■

Here we must note that Lemma 3 allows us to replace an optimization problem with a non-convex cost function and a rank-one constraint, by a problem with a convex (for $\alpha \geq 0$) cost function and two rank-one constraints. Obviously, these constraints still make the problem non-convex and very difficult to solve. In particular, it can be proved that in the MaxMin case (i.e., $\alpha = \infty$), and with the relaxation of the rank one constraint on $\mathbf{W}$, the above optimization problem is mathematically identical to the MaxMin fair multicast beamforming problem [25], [26], which has been proved to be NP-hard. Therefore, we can conclude that in general, our optimization problem is at least as difficult as a NP-hard problem, which precludes obtaining an optimal algorithm with affordable computational complexity.

In order to obtain an approximated solution, we can drop the two rank-one constraints in (13), which yields the following relaxed problem[7]

$$
\begin{aligned}
\underset{\tilde{\mathbf{W}},\gamma_1,\ldots,\gamma_{N_c}}{\arg\min} \quad & \frac{1}{\alpha-1}\sum_{k=1}^{N_c}(1+\gamma_k)^{1-\alpha}, \qquad (14)\\
\text{subject to} \quad & \gamma \mathrm{Tr}(\tilde{\mathbf{H}}_k\tilde{\mathbf{W}}) = \gamma_k, \qquad k=1,\ldots,\mathbf{N}_c\\
& \mathrm{Tr}(\tilde{\mathbf{W}}) = 1,\\
& \tilde{\mathbf{W}} \succeq \mathbf{0}.
\end{aligned}
$$

Now, for $\alpha \geq 0$, the problem in (14) is a convex optimization problem, which can be solved by means of standard techniques. In general, however, the solution $\tilde{\mathbf{W}}$ of (14) will not satisfy the original rank-one constraints, and we will have to generate an approximated rank-one solution to (13) from $\tilde{\mathbf{W}}$. A common technique in optimization consists in a randomization process (see [25], [26] and the references therein), which generates several candidate solutions and selects the one providing minimum cost.

Although the above formulation allows us to obtain an approximated solution to the original problem by means of SDR techniques, in general the computational complexity of the overall algorithm can be prohibitive for practical applications. As an example, let us consider a practical MIMO system such as that used in the simulations ($N_c = 64$ data carriers and $n_T = n_R = 4$ antennas). In this case, we have $N_c = 64$ slack variables and a 16×16 positive semidefinite matrix $\tilde{\mathbf{W}}$. Thus, even assuming a linear cost function, the computational cost of the problem in (14) can be as large as $\mathcal{O}\left((N_c + n_T^2 n_R^2)^{3.5}\right) \approx 5 \cdot 10^8$ [25], [26]. Furthermore, the application of a randomization method with a sufficient number of candidates (see [25], [26] for typical numbers of randomizations used in a moderate-sized problem) would also result in a prohibitive computational burden.

D. Proposed Beamforming Algorithm

In order to avoid the computational cost associated to the SDR approach, we propose a simple iterative algorithm which, equipped with an adequate initialization point that can be obtained in closed form, provides good results in most practical cases. Let us start by briefly describing the initialization method, which obtains an approximated MaxSNR solution in closed form. In particular, for $\alpha = 0$ (or $\gamma \simeq 0$) the optimization problem in (5) can be rewritten as

$$
\begin{aligned}
\underset{\mathbf{W},\mathbf{w}}{\arg\max} \quad & \sum_{k=1}^{N_c} \mathbf{w}^H \tilde{\mathbf{H}}_k \mathbf{w},\\
\text{subject to} \quad & \|\mathbf{w}\| = 1,\\
& \mathrm{vec}(\mathbf{W}) = \mathbf{w},\\
& \mathrm{rank}(\mathbf{W}) = 1.
\end{aligned}
$$

Thus, defining $\mathbf{R} = \sum_{k=1}^{N_c} \tilde{\mathbf{H}}_k$ and relaxing the rank-one constraint we obtain

$$
\underset{\mathbf{w}}{\arg\max} \quad \mathbf{w}^H \mathbf{R} \mathbf{w}, \qquad \text{subject to} \qquad \|\mathbf{w}\| = 1,
$$

whose solution is given by the principal eigenvector of $\mathbf{R}$. As previously pointed out, the matrix $\mathbf{W}$ obtained from the solution $\mathbf{w} = \mathbf{v}_{\max}(\mathbf{R})$ will not be rank-one in general, and we will have to apply a randomization step or a similar approach. Here, we propose a simpler alternative, which obtains the best (in the squared-norm sense) rank-one approximation of $\mathbf{W}$, i.e., we obtain the transmit and receive beamformers as the left and right singular vectors of $\mathbf{W}$.

After obtaining the initialization point, the proposed iterative algorithm is based on the following updating rules

$$
\mathbf{w}_T(t+1) = \mathbf{w}_T(t) + \mu \mathbf{R}_{\mathrm{MISO}_\alpha}(t)\mathbf{w}_T(t), \qquad (15)
$$

$$
\mathbf{w}_R(t+1) = \mathbf{w}_R(t) + \mu \mathbf{R}_{\mathrm{SIMO}_\alpha}(t)\mathbf{w}_R(t), \qquad (16)
$$

where μ is a step-size (or regularization parameter) and t denotes the iteration index. The above expressions, which can be seen as a simple gradient search algorithm, are inspired by the coupled EV problems in (6), and they can be interpreted as iterations of a power method for obtaining the solution of (6).[8] Specifically, the power method is applied to the regularized matrices $\mathbf{I} + \mu \mathbf{R}_{\mathrm{MISO}_\alpha}(t)$ and $\mathbf{I} + \mu \mathbf{R}_{\mathrm{SIMO}_\alpha}(t)$, where the regularization factor avoids convergence problems due to large variations of the matrices between consecutive iterations. Thus, the overall technique, which includes a normalization step to force the unit energy constraint on the beamformers, is summarized in Algorithm 1.

Regarding the computational complexity, it is easy to find that the initialization step has a complexity of order $\mathcal{O}(n_T^3 n_R^3 + N_c n_T^2 n_R^2)$, whereas one iteration of the proposed method comes at a cost of approximately $\mathcal{O}\left(N_c(n_T + n_R)^2\right)$. Thus, 50 iterations of the proposed algorithm in the previous example ($N_c = 64$ and $n_T = n_R = 4$) would have a cost three orders of magnitude lower than that of the SDR approach previous to the randomization technique.

Finally, analogously to other iterative techniques, the proposed algorithm can suffer from local minima. Nevertheless, we have verified by means of numerous simulations that, thanks to the initialization in the approximated MaxSNR solution, the proposed method provides very satisfactory results in most cases. Additionally, although it is beyond the scope of this paper, we must note that the convergence speed of the proposed algorithm could be further improved by adaptively changing the learning rate μ, and that the algorithm can be easily modified to obtain a graduated nonconvexity technique [29]. In particular, we can make a smooth transition from the initialization point ($\alpha = 0$ or $\gamma \simeq 0$) to the desired value of α and γ (see [30] for an application of this idea in the context of sparse representations).

V. Simulation Results

The performance of the proposed technique is illustrated in this section by means of Monte Carlo simulations. In all the experiments, we consider a MIMO system with 64 subcarriers and $n_T = n_R = 4$ transmit and receive antennas. An i.i.d. Rayleigh MIMO channel model with exponential

[7]Note that the matrix $\mathbf{W}$ can be removed because it only appears in the constraint $\mathrm{vec}(\mathbf{W}) = \mathbf{v}_{\max}(\tilde{\mathbf{W}})$.

[8]The proposed iterative approach is also closely related to alternating minimization methods [5], [7], [28].

Select μ and α; initialize $\mathbf{w}_T$ and $\mathbf{w}_R$.
repeat
  Update of the transmit beamformer
  Obtain the equivalent MISO channels $\mathbf{h}_{\text{MISO}_k}$ with (9).
  Update h_k and MSE_k for $k = 1, \ldots, N_c$.
  Obtain the matrix $\mathbf{R}_{\text{MISO}_\alpha}$ with (7).
  Update the beamformer $\mathbf{w}_T$ with (15).
  Normalize the solution: $\mathbf{w}_T = \mathbf{w}_T / \|\mathbf{w}_T\|$.
  Update of the receive beamformer
  Obtain the equivalent SIMO channels $\mathbf{h}_{\text{SIMO}_k}$ with (9).
  Update h_k and MSE_k for $k = 1, \ldots, N_c$.
  Obtain the matrix $\mathbf{R}_{\text{SIMO}_\alpha}$ with (8).
  Update the beamformer $\mathbf{w}_R$ with (16).
  Normalize the solution: $\mathbf{w}_R = \mathbf{w}_R / \|\mathbf{w}_R\|$.
until Convergence

Algorithm 1: Proposed beamforming algorithm.

power delay profile has been assumed. In particular, the total power associated to the l-th tap is

$$E\left[\|\mathbf{H}[l]\|^2\right] = (1-\rho)\rho^l n_T n_R, \qquad l = 0, \ldots, L_c - 1,$$

where L_c is the length of the channel impulse response ($L_c = 16$ in the simulations), and the exponential parameter ρ has been selected as $\rho = 0.7$. We have focused on the MaxSNR ($\alpha = 0$) [4]–[7], [12], [23], MaxCAP ($\alpha = 1$) and MinMSE ($\alpha = 2$) approaches, which have been compared with a SISO system and with a full MIMO scheme applying maximum ratio transmission (MRT) and maximum ratio combining (MRC) per subcarrier (denoted as Full-MIMO), which can be seen as an upper bound for the performance of any analog antenna combining system.

Each Monte Carlo simulation consists in the generation of a channel realization, the obtention of the transmit and receive beamformers, and the evaluation of the system performance, which can be based on the analysis of the equivalent SISO channel, or the transmission of one OFDM symbol. In all the examples, we have performed a minimum of 10000 Monte Carlo simulations. However, in those experiments involving very low outage probabilities or BER values, the number of simulations has been increased to guarantee a minimum of 10 outage situations (or 10 incorrectly decoded OFDM symbols).

In all the experiments, the step-size has been fixed to $\mu = 0.1$, and the convergence criterion is based on the difference between the beamformers in two consecutive iterations. Specifically, the algorithm finishes when the Euclidian distance is lower than 10^{-3}. With these values, the proposed algorithm has never[9] exceeded 50 iterations. As an example, the convergence of the MaxSNR, MaxCAP and MinMSE algorithms for a SNR of 10 dB is illustrated in Fig. 3. As can be seen, with the initialization in the approximated MaxSNR beamformers, the proposed algorithm converges very fast to the desired solution.

A. Equivalent Channel Properties

In the first set of examples we analyze the equivalent channel after beamforming for a fully loaded system ($N_c = 64$).

[9]Note that, in the cases of low BERs or outage probabilities, we have performed several millions of Monte Carlo simulations.

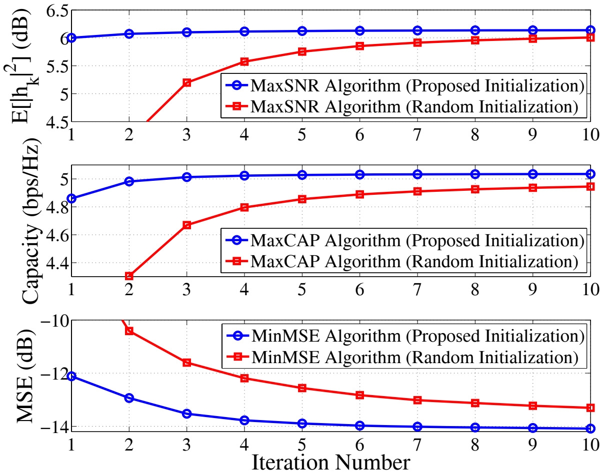

Fig. 3. Convergence of the MaxSNR, MaxCAP and MinMSE algorithms for SNR=10 dB. Initialization in the approximated MaxSNR solution or in a pair of unit-norm random vectors $\mathbf{w}_T$, $\mathbf{w}_R$.

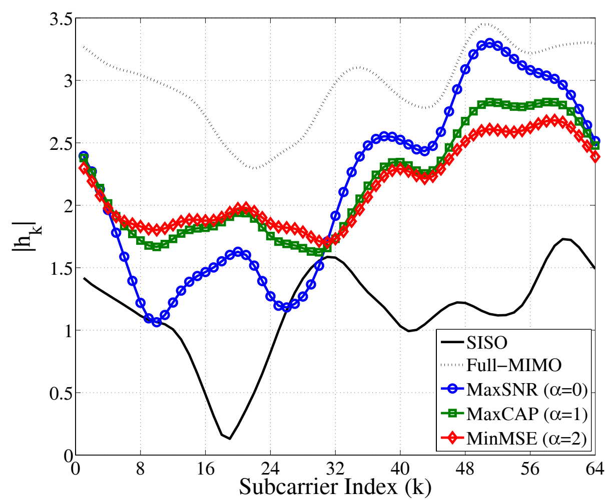

Fig. 4. Channel response $|h_k|$ after beamforming for a channel realization.

Fig. 4 shows the frequency response of the equivalent channel for a random channel realization and a SNR $\gamma = 10$ dB. As can be seen, the parameter α establishes a tradeoff between the energy and the spectral flatness of the equivalent channel. Furthermore, as expected, the performance of the proposed analog combining schemes is between that of the SISO and Full-MIMO systems.

This effect can be seen more clearly in Fig. 5, which shows the probability density function (obtained from 10000 random channel realizations) of the squared amplitude of the equivalent channel for a SNR of $\gamma = 10$ dB. As can be seen, the MaxCAP and MinMSE approaches avoid values close to zero at the expense of a slight degradation of the overall SNR. Finally, Figs. 6 and 7 show the outage probability for a capacity of 5 bps/Hz and the evolution of the total MSE with the SNR. As expected, the best results are provided by the MaxCAP and MinMSE approaches, respectively, whereas

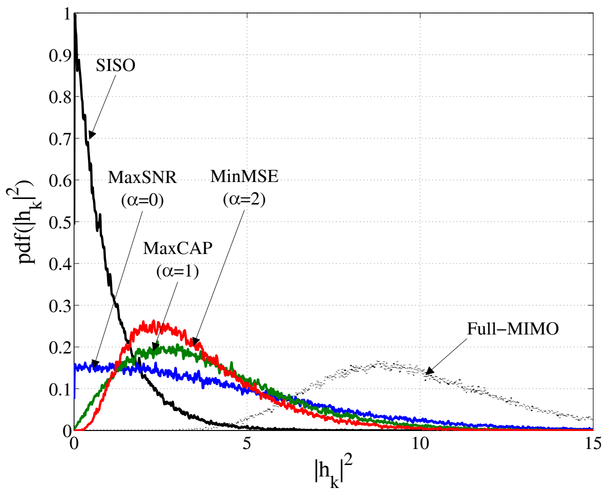

Fig. 5. Probability density function (pdf) of the equivalent channel response $|h_k|^2$. $\gamma = 10\,\text{dB}$.

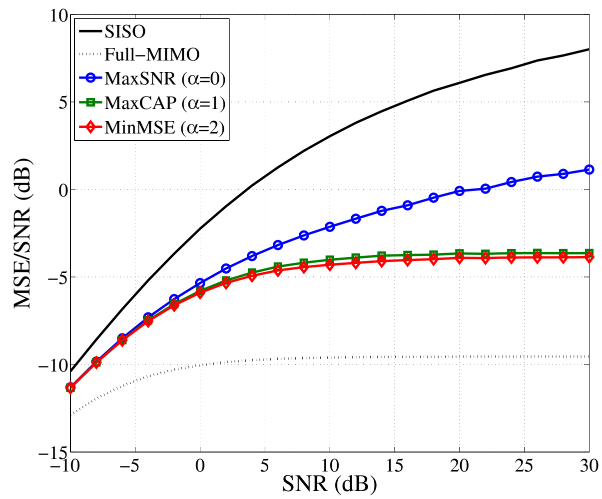

Fig. 7. Evolution of the total MSE with the SNR.

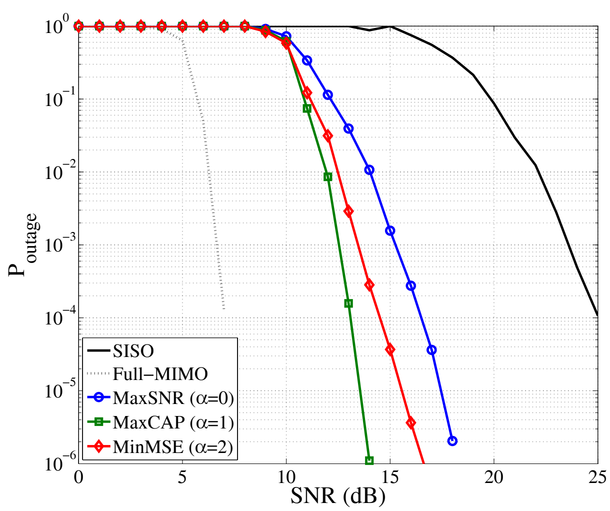

Fig. 6. Outage probability for a transmission rate of 5 bps/Hz.

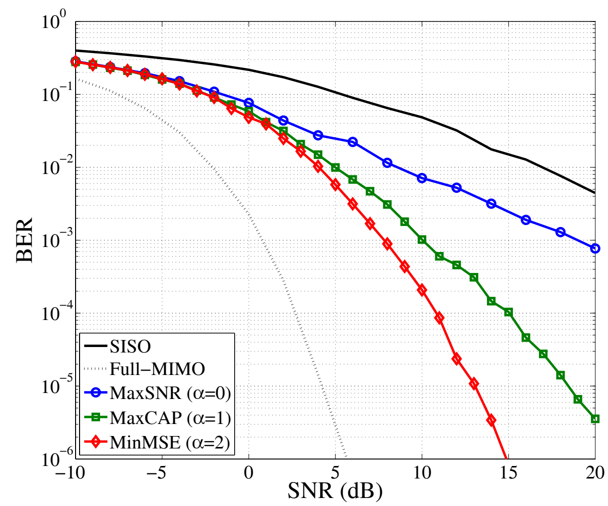

Fig. 8. Bit error rate for the proposed criteria. Uncoded QPSK symbols.

the MaxSNR criterion suffers significant performance degradations.

B. Uncoded Transmissions

The advantage of the proposed MaxCAP and MinMSE criteria over the MaxSNR approach becomes clearer when the system performance is evaluated in terms of BER. Fig. 8 shows the BER for uncoded QPSK transmissions with $N_c = 64$ data carriers and LMMSE receivers. As can be seen, the MinMSE approach outperforms the remaining analog combining criteria, which is due to the fact that, for uncoded transmissions, the overall system performance is dominated by the worst data carriers. Therefore, since the MinMSE criterion assigns the highest weights (MSE_k^2) to these critical carriers, it provides better results than those of the MaxCAP and MaxSNR approaches.

Finally, Fig. 9 shows the BER when $N_c = 64$ QPSK symbols are linearly precoded with the FFT matrix and the receiver is based on the LMMSE criterion. In this case the minimization of the BER is equivalent to the minimization of the MSE [9], [24], which explains the good performance of the MinMSE beamforming criterion.

C. Coded Transmissions

In this pair of examples, the proposed schemes have been evaluated in a more practical situation. In particular, we have adopted the 802.11a standard [31], which uses $N_c = 48$ out of 64 subcarriers for data transmission. The information bits are encoded with a convolutional code and block interleaved as specified in the standard. Finally, the receiver is based on a soft Viterbi decoder.

In the first example we have selected a transmission rate of 12 Mbps, which implies QPSK signaling and a convolutional code of rate 1/2. Here, the introduction of a channel encoder could induce us to think that the MaxCAP criterion will outperform the remaining approaches. However, as can be

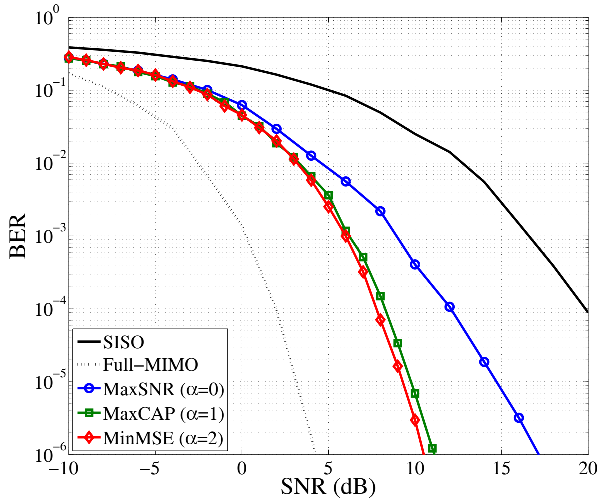

Fig. 9. Bit error rate for the proposed criteria. QPSK symbols linearly precoded with the FFT matrix.

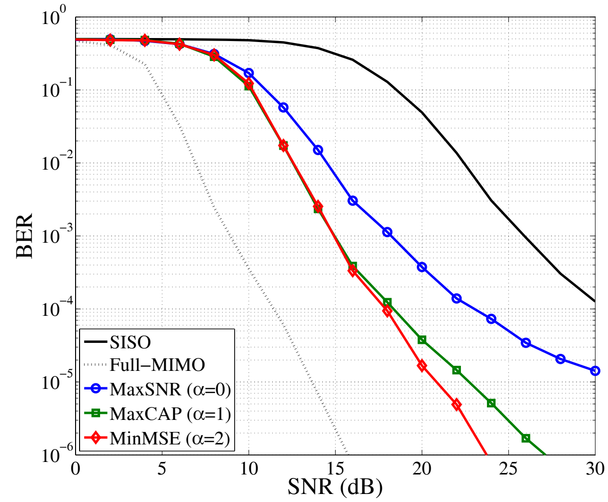

Fig. 11. BER for a 802.11a based system with transmission rate of 54 Mbps. 64QAM signaling and convolutional encoder of rate 3/4.

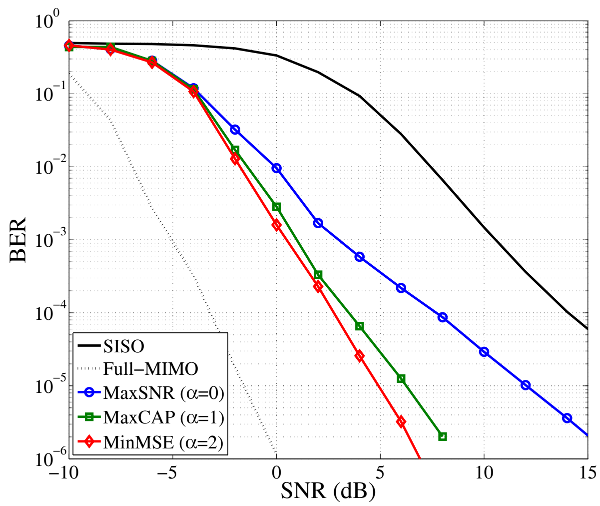

Fig. 10. BER for a 802.11a based system with transmission rate of 12 Mbps. QPSK signaling and convolutional encoder of rate 1/2.

seen in Fig. 10, the best results are again provided by the MinMSE beamformers. This is due to the fact that we are not using an ideal channel encoder (note that the channel encoder operates on an OFDM symbol basis), which implies that a slight degradation in the capacity can be acceptable in order to obtain a less frequency selective equivalent channel. Finally, the same conclusions can be reached from the experiment with transmission rate of 54 Mbps (64 QAM signaling and 3/4 convolutional encoder), whose results are shown in Fig. 11.

D. Effects of RF Impairments and Channel Estimation Errors

In the final example we have included RF impairments and channel estimation errors. In particular, we have obtained least-squares (LS) estimates of $\mathbf{H}_k$ $(k = 1, \ldots, N_c)$ and σ^2 by means of the sequential transmission (using different pairs of orthogonal transmit and receive beamformers) of $n_T n_R = 16$ training OFDM symbols. The estimated channel and noise variance have been used to obtain the transmit and receive beamformers, which are quantified with a resolution of 5 bits. Additionally, due to RF impairments, there exist a small difference between the quantized weights and the actual values applied in each antenna. This error is modeled as an i.i.d. uniform noise with the same range as that of the quantization error. The obtained results for the MinMSE ($\alpha = 2$) case and 802.11a coded transmissions with a rate of 12Mbps are shown in Fig. 12, where we can see that the realistic RF-combining system clearly outperforms the idealized SISO system. Furthermore, its performance degradation with respect to an idealized RF-combining system is of approximately 3 dB. Finally, we have verified by means of simulations that the 3 dB gap is mainly due to the effect of the channel estimation errors in the decoding process, and not to the small errors in the beamformers. Therefore, we can conclude that similar degradations would take place in a pre-FFT based system with channel estimation errors.

VI. Conclusions

In this paper we have proposed a general beamforming criterion for a novel MIMO transceiver, which performs adaptive signal combining in the RF domain. With this new combining architecture and under multicarrier transmissions, the same pair of Tx-Rx beamformers must be applied to all the subcarriers and, due to this coupling, the beamforming design problem poses several new challenges in comparison to conventional MIMO schemes. Considering the case of perfect channel state information at the receiver side, we have proposed a beamforming criterion which depends on a single parameter α. This parameter establishes a tradeoff between the energy and spectral flatness of the equivalent channel, and allows us to obtain some interesting design criteria. In particular, the proposed beamforming criterion can be reduced to the maximization of the received SNR (MaxSNR, $\alpha = 0$),

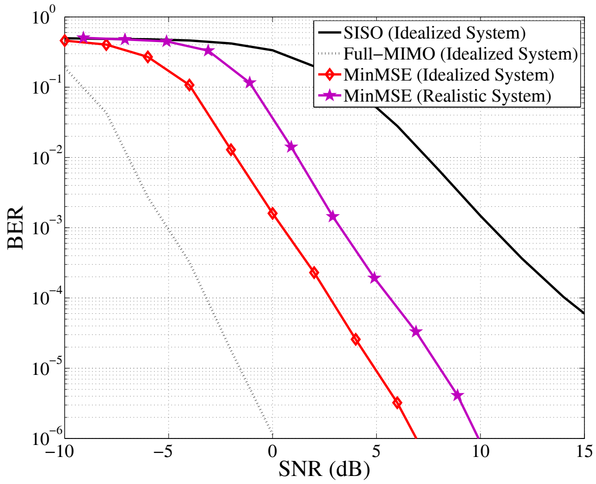

Fig. 12. Performance of the MinMSE criterion in a 802.11a based system with transmission rate of 12 Mbps including the effect of channel estimation errors and RF impairments.

the maximization of the system capacity (MaxCAP, $\alpha = 1$), and the minimization of the MSE (MinMSE, $\alpha = 2$) of the optimal linear receiver. In general, the proposed criterion results in a non-convex optimization problem. We have shown that the noncovexity essentially comes from two rank-one constraints that, when removed, allow us to apply SDR techniques. Furthermore, to avoid the high computational cost of SDR techniques, we have proposed a simple and efficient algorithm which, with a proper initialization, provides very good results in practical OFDM-based WLAN standards such as 802.11a. Finally, the numerous simulation results allow us to conclude that, in general, it is a good idea to increase the spectral flatness of the equivalent SISO channel, even at the expense of a slight degradation in the overall SNR.

Appendix
Proof of Property 3

A. Rewriting the Cost Function

Let us start by writing $\text{MSE}_k = (P_\beta p_{\beta,k})^{\frac{1}{\beta-1}}$. Thus, the cost function $f_\alpha(\mathbf{w}_T, \mathbf{w}_R)$ can be rewritten as

$$f_\alpha(\mathbf{w}_T, \mathbf{w}_R) = \frac{1}{\alpha-1} \log\left(\frac{1}{N_c} \sum_{k=1}^{N_c} (P_\beta p_{\beta,k})^{\frac{\alpha-1}{\beta-1}} \right),$$

and after a straightforward manipulation we obtain

$$f_\alpha(\mathbf{w}_T, \mathbf{w}_R) = f_\beta(\mathbf{w}_T, \mathbf{w}_R) + g_{\alpha,\beta}\left(\mathbf{p}_\beta(\mathbf{w}_T, \mathbf{w}_R)\right), \quad (17)$$

with

$$g_{\alpha,\beta}(\mathbf{p}_\beta(\mathbf{w}_T, \mathbf{w}_R)) = \frac{\alpha-\beta}{(\alpha-1)(\beta-1)} \log(N_c) + \frac{1}{\alpha-1} \log\left(\sum_{k=1}^{N_c} p_{\beta,k}^{\frac{\alpha-1}{\beta-1}} \right).$$

Eq. (17) shows that the cost function for a given parameter α can be written as the cost function for a different parameter β, plus a penalty term that depends on α, β and $p_{\beta,k}$.

B. Analysis of the Penalty Term

1) Antisymmetry: From eq. (17), and interchanging the values α and β, we can directly conclude that

$$g_{\alpha,\beta}\left(\mathbf{p}_\beta\right) = -g_{\beta,\alpha}\left(\mathbf{p}_\alpha\right),$$

where, for notational simplicity, we have omitted the dependency with the beamformers. Thus, we can restrict our study to the case $\alpha > \beta$.

2) Monotonicity: In order to prove that $g_{\alpha,\beta}\left(\mathbf{p}_\beta\right)$ increases with α we evaluate its derivative

$$\frac{\partial g_{\alpha,\beta}\left(\mathbf{p}_\beta\right)}{\partial \alpha} = \frac{\log(N_c)}{(\alpha-1)^2} + \frac{\sum_{k=1}^{N_c} p_{\beta,k}^{\frac{\alpha-1}{\beta-1}} \log\left(p_{\beta,k}^{\frac{\alpha-1}{\beta-1}}\right)}{(\alpha-1)^2 \sum_{k=1}^{N_c} p_{\beta,k}^{\frac{\alpha-1}{\beta-1}}} - \frac{\log\left(\sum_{k=1}^{N_c} p_{\beta,k}^{\frac{\alpha-1}{\beta-1}}\right)}{(\alpha-1)^2},$$

which, after defining $A = \sum_{k=1}^{N_c} p_{\beta,k}^{\frac{\alpha-1}{\beta-1}}$ and $a_k = \frac{p_{\beta,k}^{\frac{\alpha-1}{\beta-1}}}{A}$, can be rewritten as

$$\frac{\partial g_{\alpha,\beta}\left(\mathbf{p}_\beta\right)}{\partial \alpha} = \frac{1}{(\alpha-1)^2} \sum_{k=1}^{N_c} a_k \log(a_k N_c).$$

Thus, taking into account that $\sum_{k=1}^{N_c} a_k = 1$ (i.e., a_k can be seen as the probability mass function of a discrete random variable), it is easy to prove that the above function is Schur-convex with respect to a_k ($k = 1, \ldots, N_c$) [9], [11], i.e., it attains its minimum value for $a_k = \frac{1}{N_c}$, which yields $\frac{\partial g_{\alpha,\beta}(\mathbf{p}_\beta)}{\partial \alpha} = 0$. Therefore, we can conclude that the derivative is non-negative and the function $g_{\alpha,\beta}\left(\mathbf{p}_\beta\right)$ increases with α.

3) Schur-Convexity of $g_{\alpha,\beta}\left(\mathbf{p}_\beta\right)$: The proof is based on the two following observations. Firstly, for $\alpha > \beta$, the function

$$\sum_{k=1}^{N_c} p_{\beta,k}^{\frac{\alpha-1}{\beta-1}},$$

is Schur-convex if $\alpha > 1$ and Schur-concave if $\alpha < 1$ [9], [11]. Secondly, the function $\frac{1}{\alpha-1}\log(\cdot)$ is increasing for $\alpha > 1$ and decreasing for $\alpha < 1$. Thus, $\forall \alpha > \beta$, the composite function $g_{\alpha,\beta}\left(\mathbf{p}_\beta\right)$ is Schur-convex [9], [11], which implies that it attains its minimum when

$$p_{\beta,k} = \frac{1}{N_c}, \qquad \text{for } k = 1, \ldots, N_c.$$

Finally, when all the $p_{\beta,k}$ are equal we have that $g_{\alpha,\beta}\left(\mathbf{p}_\beta\right) = 0$, which implies that $g_{\alpha,\beta}\left(\mathbf{p}_\beta\right)$ is non-negative for $\alpha > \beta$.

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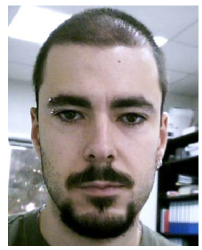

Javier Vía (M'08) received his Telecommunication Engineer Degree and his Ph.D. in electrical engineering from the University of Cantabria, Spain in 2002 and 2007, respectively. In 2002 he joined the Department of Communications Engineering, University of Cantabria, Spain, where he is currently Assistant Professor. He has spent visiting periods at the Smart Antennas Research Group of Stanford University, and at the Department of Electronics and Computer Engineering (Hong Kong University of Science and Technology). Dr. Vía has actively participated in several European and Spanish research projects. His current research interests include blind channel estimation and equalization in wireless communication systems, multivariate statistical analysis, quaternion signal processing and kernel methods.

Ignacio Santamaría (M'96-SM'05) received his Telecommunication Engineer Degree and his Ph.D. in electrical engineering from the Universidad Politécnica de Madrid (UPM), Spain, in 1991 and 1995, respectively. In 1992 he joined the Departamento de Ingeniería de Comunicaciones, Universidad de Cantabria, Spain, where he is currently Full Professor. He has been a visiting researcher at the Computational NeuroEngineering Laboratory (University of Florida), and at the Wireless Networking and Communications Group (University of Texas at Austin). He has more than 100 publications in refereed journals and international conference papers. His current research interests include signal processing algorithms for wireless communication systems, MIMO systems, multivariate statistical techniques and machine learning theories. He has been involved in several national and international research projects on these topics. He is currently serving as a member of the Machine Learning for Signal Processing Technical Committee of the IEEE Signal Processing Society.

Víctor Elvira (S'08) received the Bachelor degree in engineering telecommunications and the Master degree from the University of Cantabria, Spain in 2007 and 2008, respectively. In 2008 he joined the Department of Communications Engineering, University of Cantabria, Spain, where he is currently working towards his Ph.D. Víctor Elvira is involved in the European Union funded project MIMAX (2008-2010).

Ralf Eickhoff (S'04-M'06) received the Diploma degree in electrical engineering and the Ph.D. degree from the University of Paderborn, Germany, in 2003 and 2007, respectively. From 2003 until 2006, he held a scholarship in a graduate college "Automatic Configuration in Open Systems." During this time, he was a research assistant involved with adaptive ICs and systems. Since 2006, he has been with the Technische Universitaet Dresden, Germany, where he is currently a postdoctoral researcher involved with local positioning and wireless communication systems. His main research interests are low-power circuits and adaptive antenna combining in wireless communication systems. He was the coordinator of the European Union (EU) funded project RESOLUTION (2006-2009) and currently coordinates the EU project MIMAX (2008-2010).