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# Analog Antenna Combining for Maximum Capacity under OFDM Transmissions

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***Abstract*—In this paper, we study beamforming schemes for a novel MIMO transceiver, which performs adaptive signal combining in the radio-frequency domain. Assuming perfect channel knowledge at both the transmit and receive sides, we consider the problem of selecting the transmit and receive RF beamformers that maximize the capacity (MaxCAP criterion) of the system under orthogonal frequency division multiplexing (OFDM) transmissions. This problem is non-convex and has no closed-form solution, therefore the maximum capacity beamformers are found using a gradient search algorithm. Furthermore, it is shown in the paper that, for low signal-to-noise ratios (SNR), the MaxCAP criterion is equivalent to maximizing the received SNR (MaxSNR criterion). However, for moderate and high SNRs, the maximum capacity beamformers sacrifice part of the received SNR in order to improve the worst subcarriers and, in this way, they increase the overall capacity of the multicarrier channel. Finally, by means of numerical examples we show that the MaxCAP criterion significantly outperforms the MaxSNR criterion in terms of bit error rate and outage probability.**

## I. Introduction

To exploit the benefits (e.g., diversity or multiplexing gain) of multiple-input multiple-output (MIMO) wireless communication systems all antenna paths must be independently acquired and processed at base band. Consequently, the hardware costs, size and power consumption of conventional MIMO systems are increased accordingly. These higher costs explain in part the delay in the commercial deployment of multiple-antenna wireless transceivers, mainly in handsets or small cost terminals.

A RF-MIMO receiver architecture, shown in Fig. 1, solves some of these problems by shifting spatial signal processing from the base band to the radio-frequency (RF) front-end. The RF-MIMO transmitter operates analogously. The basic idea consists of performing adaptive signal combining in the radio frequency domain. In this way, after combining the RF signals a single stream of data must be acquired and processed and thus the hardware cost and the power consumption are significantly reduced [1].

It has been shown in [2] that, although the multiplexing gain of the RF-MIMO transceiver is limited to one (since we transmit/receive a single data stream), other benefits of the MIMO channel such as full spatial diversity or full array gain can be retained by the proposed architecture if proper processing is carried out.

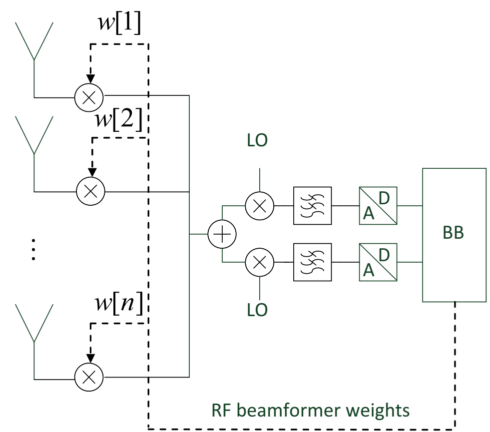


Fig. 1. Analog antenna combining in the RF path for MIMO communications systems. Exemplarily shown for a direct-conversion receiver.

From a signal processing point of view, the adaptive antenna combining architecture in Fig. 1 poses several challenging design problems. Specifically, in this paper we address the problem of selecting the RF weights (or beamformers) under OFDM transmissions with perfect channel state information at the transmitter (CSIT) and receiver (CSIR). While a conventional OFDM-MIMO receiver computes the fast Fourier transform (FFT) of each base band signal, and hence it can apply the optimal processing independently for each subcarrier; the new RF-MIMO transceiver uses the same pair of beamformers for all the subcarriers and therefore the problem is inherently coupled.

To the best of our knowledge, all the published works on this problem have only considered the maximization of the received SNR as the criterion to find the optimal beamformers [3]–[7]. As an alternative, we propose to find the beamformers that maximize the system capacity (MaxCAP criterion). Similarly to the MaxSNR criterion in the MIMO case, the MaxCAP problem is non-convex and has no closed-form solution. However, we derive a pair of coupled eigenvalue problems which must be fulfilled by the maximum capacity beamformers, and which can be efficiently solved by a simple gradient descent algorithm.

Interestingly, in the low SNR regime the MaxCAP and the MaxSNR criteria are equivalent. However, for moderate and high SNRs the proposed criterion sacrifices part of the received SNR in order to improve the equivalent channel of the worst subcarriers, thus significantly improving the outage capacity and the bit error rate in comparison to the MaxSNR criterion for both coded and uncoded transmissions.

## II. RF Beamforming under OFDM transmissions

### A. Notation and Data Model

Throughout this paper we will use bold-faced upper case letters to denote matrices, e.g., $\mathbf{X}$, bold-faced lower case letters for column vector, e.g., $\mathbf{x}$, and light-faced lower case letters for scalar quantities. The superscripts $(\cdot)^T$, $(\cdot)^H$, $(\cdot)^*$ denote transpose, Hermitian and complex conjugate. $\|\mathbf{A}\|$ and $\text{vec}(\mathbf{A})$ will denote the Frobenius norm and the column-wise vectorized version of matrix $\mathbf{A}$. Finally, the Kronecker product and the identity matrix of the required dimensions will be denoted by $\otimes$ and $\mathbf{I}$, respectively.

### B. Problem statement

Let us consider a RF-MIMO system with $n_T$ transmit and $n_R$ receive antennas, and with unit-energy transmit and receive beamformers $\mathbf{w}_T \in \mathbb{C}^{n_T \times 1}$ and $\mathbf{w}_R \in \mathbb{C}^{n_R \times 1}$ defined by the RF weights in Fig. 1. Assuming a transmission scheme based on OFDM with $N_c$ subcarriers and using a cyclic prefix longer than the channel impulse response, the communication system after TX/RX radio frequency beamforming may be decomposed into a set of parallel and non-interfering single-input single-output (SISO) equivalent channels given by

$$y_k[n] = h_k s_k[n] + n_k[n], \qquad k = 1, \ldots, N_c,$$

where $y_k[n]$, $s_k[n]$ and $n_k[n]$ are, respectively, the observation, transmitted signal and noise associated to the $k$-th subcarrier of the $n$-th OFDM symbol, and the equivalent flat-fading SISO channels are

$$h_k = \mathbf{w}_R^H \mathbf{H}_k \mathbf{w}_T, \qquad k = 1, \ldots, N_c, \tag{1}$$

where $\mathbf{H}_k \in \mathbb{C}^{n_R \times n_T}$ represents the MIMO channel for the $k$-th subcarrier.[1]

Conventional MIMO-OFDM base band schemes have access to the signals at each one of the transmitting/receiving antennas and, consequently, can obtain a different pair of beamformers per subcarrier. This scheme is sometimes called post-FFT processing [7] and we also refer to it as full MIMO processing. However, with the novel RF combining architecture a per-carrier beamforming is not possible since all the orthogonal MIMO channels $\mathbf{H}_k$ are affected by the same pair of beamformers. This coupling among the different MIMO channels imposes some trade-offs and represents the main challenge for the design of the beamformers.

[1]In this paper we assume perfect knowledge of $\mathbf{H}_k$. However, we must point out that the estimation and tracking of the MIMO channel is a challenging problem which will be addressed in future works.

## III. Previous Work: MaxSNR Beamforming

As previously commented, most works addressing analog combining architectures [3], or pre-DFT schemes [4], [5], [7], find the beamformers that maximize the SNR of the received signal. This MaxSNR criterion is equivalent to maximizing the energy of the equivalent SISO channel, i.e.,

$$\underset{\substack{\mathbf{w}_T, \mathbf{w}_R \\ \|\mathbf{w}_T\| = \|\mathbf{w}_R\| = 1}}{\arg\max} \sum_{k=1}^{N_c} |h_k|^2 = \sum_{k=1}^{N_c} |\mathbf{w}_R^H \mathbf{H}_k \mathbf{w}_T|^2. \tag{2}$$

The optimization problem (2) can be solved in closed form for the particular cases of single-input multiple-output (SIMO) or multiple-input single-output (MISO) channels. However, in the general case of MIMO channels the problem is more complicated and no closed-form solution can be found. Typically, the MaxSNR problem in the MIMO case has been solved using iterative techniques such as that proposed in [5], which consists in alternate maximizations of the received SNR for the equivalent SIMO and MISO channels, i.e., at each iteration the transmit (receive) beamformer is considered fixed, and the optimum $\mathbf{w}_R$ ($\mathbf{w}_T$) for the equivalent SIMO (MISO) channel is obtained as the principal eigenvector of the matrix [3]–[5]

$$\mathbf{R}_{\text{SIMO}}^{\text{SNR}} = \sum_{k=1}^{N_c} \mathbf{h}_{\text{SIMO}_k} \mathbf{h}_{\text{SIMO}_k}^H, \quad \mathbf{R}_{\text{MISO}}^{\text{SNR}} = \sum_{k=1}^{N_c} \mathbf{h}_{\text{MISO}_k} \mathbf{h}_{\text{MISO}_k}^H, \tag{3}$$

where

$$\mathbf{h}_{\text{SIMO}_k} = \mathbf{H}_k \mathbf{w}_T, \qquad \mathbf{h}_{\text{MISO}_k} = \mathbf{H}_k^H \mathbf{w}_R, \tag{4}$$

are the equivalent SIMO (MISO) channels after fixing the transmit (receive) beamformer.[2]

## IV. Proposed Method: MaxCAP Beamforming

As an alternative to the MaxSNR criterion in this section we propose the MaxCAP criterion, which maximizes the capacity of the equivalent multicarrier SISO channel given by (1). First, we formulate the optimization problem and derive a set of coupled eigenvalue problems that must be fulfilled by the optimal TX/RX beamformers. Second, a simple gradient descent technique, equipped with a good initialization technique, is proposed to find the solution of this nonlinear set of equations. Finally, some insights about the behavior of the method and its relationship with the MaxSNR criterion in low SNR scenarios are discussed.

### A. MaxCAP cost function

Let us start by assuming unit power transmissions ($E\left[|s_k[n]|^2\right] = 1$) and writing the capacity of the equivalent SISO channel as

$$C(\mathbf{w}_T, \mathbf{w}_R) \propto \sum_{k=1}^{N_c} \log\left(1 + \gamma |h_k|^2\right),$$

[2]Note that in the case of SIMO/MISO systems, the vectors in (4) represent the physical channel.

where $\gamma = 1/E\left[|n_k[n]|^2\right]$ is the averaged SNR for all the subcarriers.[3] Then, the MaxCAP optimization criterion can be written as

$$\underset{\substack{\mathbf{w}_T,\mathbf{w}_R \\ \|\mathbf{w}_T\|=\|\mathbf{w}_R\|=1}}{\arg\max} \sum_{k=1}^{N_c} \log\left(1+\gamma|h_k|^2\right). \quad (5)$$

Solving now (5) with respect to $\mathbf{w}_R$ and $\mathbf{w}_T$ by means of the Lagrange multipliers method, we easily obtain the following pair of eigenvalue problems

$$\mathbf{R}_{\text{SIMO}}^{\text{CAP}}\mathbf{w}_R = \lambda\mathbf{w}_R, \qquad \mathbf{R}_{\text{MISO}}^{\text{CAP}}\mathbf{w}_T = \lambda\mathbf{w}_T, \quad (6)$$

where $\lambda$ is the Lagrange multiplier and the products $\mathbf{R}_{\text{SIMO}}^{\text{CAP}}\mathbf{w}_R$, $\mathbf{R}_{\text{MISO}}^{\text{CAP}}\mathbf{w}_T$ represent the gradient of $C(\mathbf{w}_T,\mathbf{w}_R)$ with respect to $\mathbf{w}_R$ and $\mathbf{w}_T$. Specifically, the SIMO/MISO matrices are given by

$$\mathbf{R}_{\text{SIMO}}^{\text{CAP}} = \sum_{k=1}^{N_c} \text{MSE}_k \mathbf{h}_{\text{SIMO}_k}\mathbf{h}_{\text{SIMO}_k}^H, \quad (7)$$

$$\mathbf{R}_{\text{MISO}}^{\text{CAP}} = \sum_{k=1}^{N_c} \text{MSE}_k \mathbf{h}_{\text{MISO}_k}\mathbf{h}_{\text{MISO}_k}^H, \quad (8)$$

where

$$\text{MSE}_k = \frac{1}{1+\gamma|h_k|^2},$$

is the mean square error associated to the $k$-th subcarrier and $\mathbf{h}_{\text{SIMO}_k}$, $\mathbf{h}_{\text{MISO}_k}$ are the equivalent SIMO (MISO) channels defined in (4).

### *B. Optimization Algorithm*

Unfortunately, the pair of eigenvalue problems in (6) are coupled through the matrices $\mathbf{R}_{\text{SIMO}}^{\text{CAP}}$ and $\mathbf{R}_{\text{MISO}}^{\text{CAP}}$, which depend on both $\mathbf{w}_T$ and $\mathbf{w}_R$, and this precludes obtaining a closed-form solution for the maximum capacity beamformers. In fact, unlike the MaxSNR criterion, closed-form MaxCAP solutions do not exist even for the simpler cases of SIMO and MISO channels. Therefore, alternating optimization approaches can neither be applied in this case.

To avoid these problems, here we propose a gradient search algorithm based on the following updating rules

$$\mathbf{w}_R = \mathbf{w}_R + \mu\mathbf{R}_{\text{SIMO}}^{\text{CAP}}\mathbf{w}_R, \quad (9)$$

$$\mathbf{w}_T = \mathbf{w}_T + \mu\mathbf{R}_{\text{MISO}}^{\text{CAP}}\mathbf{w}_T, \quad (10)$$

where $\mu$ is a given step-size. The overall method, which is summarized in Algorithm 1, can also be interpreted as a single iteration of a power method for the extraction of the eigenvectors of $\mathbf{I}+\mu\mathbf{R}_{\text{SIMO}}^{\text{CAP}}$ and $\mathbf{I}+\mu\mathbf{R}_{\text{MISO}}^{\text{CAP}}$.

Analogously to the iterative approach in [5] for solving the MaxSNR criterion, the gradient method proposed in this paper could suffer from local minima. Nevertheless, local minima can be avoided in practice by means of a proper initialization technique. In particular, we propose to initialize the algorithm using a rather accurate approximation of the MaxSNR beamformers, which can be obtained in closed-form.

[3]We assume, without loss of generality, random MIMO channels with energy $E\left[\|\mathbf{H}_k\|^2\right]=1$.

Estimate $\mathbf{H}$ and $\gamma$ and initialize $\mu$, $\mathbf{w}_R$, $\mathbf{w}_T$.
**repeat**
**Update of the receive beamformer**
Obtain the equivalent SIMO channels $\mathbf{h}_{\text{SIMO}_k}$ with (4).
Update $h_k$ and $\text{MSE}_k$ for $k=1,\dots,N_c$.
Obtain the matrix $\mathbf{R}_{\text{SIMO}}^{\text{CAP}}$ with (7).
Update the beamformer $\mathbf{w}_R$ with (9).
Normalize the solution: $\mathbf{w}_R = \mathbf{w}_R/\|\mathbf{w}_R\|$.
**Update of the transmit beamformer**
Obtain the equivalent MISO channels $\mathbf{h}_{\text{MISO}_k}$ with (4).
Update $h_k$ and $\text{MSE}_k$ for $k=1,\dots,N_c$.
Obtain the matrix $\mathbf{R}_{\text{MISO}}^{\text{CAP}}$ with (8).
Update the beamformer $\mathbf{w}_T$ with (10).
Normalize the solution: $\mathbf{w}_T = \mathbf{w}_T/\|\mathbf{w}_T\|$.
**until** Convergence

**Algorithm 1:** MaxCAP algorithm.

To find this starting point for our algorithm, notice first that the MaxSNR criterion (2) can alternatively be written as

$$\underset{\substack{\mathbf{w}=\mathbf{w}_T^*\otimes\mathbf{w}_R \\ \|\mathbf{w}\|=1}}{\arg\max} \ \mathbf{w}^H\mathbf{R}\mathbf{w}, \quad (11)$$

where

$$\mathbf{R} = \sum_{k=1}^{N_c} \text{vec}(\mathbf{H}_k)\text{vec}^H(\mathbf{H}_k).$$

Now, relaxing the constraint imposed by the Kronecker structure of $\mathbf{w}$ (which is in fact responsible for the non convexity of the problem), the optimum unconstrained vector $\tilde{\mathbf{w}}$ is given by the principal eigenvector of $\mathbf{R}$. Thus, defining $\tilde{\mathbf{W}} \in \mathbb{C}^{n_R\times n_T}$ as the matrix whose elements are taken column-wise from $\tilde{\mathbf{w}}$, we can find an approximation to the solution of (11) by minimizing

$$\left\|\mathbf{w}_R\mathbf{w}_T^H - \tilde{\mathbf{W}}\right\|^2 \qquad \text{s.t.} \qquad \|\mathbf{w}_T\| = \|\mathbf{w}_R\| = 1,$$

whose solutions $\mathbf{w}_T$ and $\mathbf{w}_R$ are directly given by the singular vectors associated to the largest singular value of $\tilde{\mathbf{W}}$. As it will be shown in the next section, with this starting point the proposed algorithm converges in a small number of iterations, and it outperforms the MaxSNR approach in terms of capacity and bit error rate.

### *C. Interpretation of the MaxCAP Solution*

From (6), it is clear that the theoretical solutions for maximum capacity are given by those eigenvectors, with associated eigenvalue $\lambda$, of the matrices $\mathbf{R}_{\text{SIMO}}^{\text{CAP}}$ and $\mathbf{R}_{\text{MISO}}^{\text{CAP}}$. Interestingly, these matrices can be seen as weighted versions, with weights given by the MSE associated to each subcarrier, of $\mathbf{R}_{\text{SIMO}}^{\text{SNR}}$ and $\mathbf{R}_{\text{MISO}}^{\text{SNR}}$ defined in (3). This implies that, in the low SNR regime ($\text{MSE}_k \simeq 1 \ \forall k$) the MaxCAP matrices coincide with their MaxSNR counterparts. Therefore, both criteria are equivalent in the low SNR regime. This can also be corroborated by taking into account the approximation

$$\log\left(1+\gamma|h_k|^2\right) \simeq \gamma|h_k|^2,$$

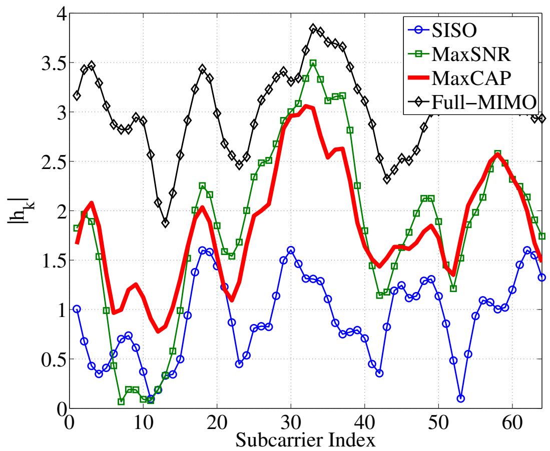


Fig. 2. Response of the equivalent SISO channel. SNR=10 dB.

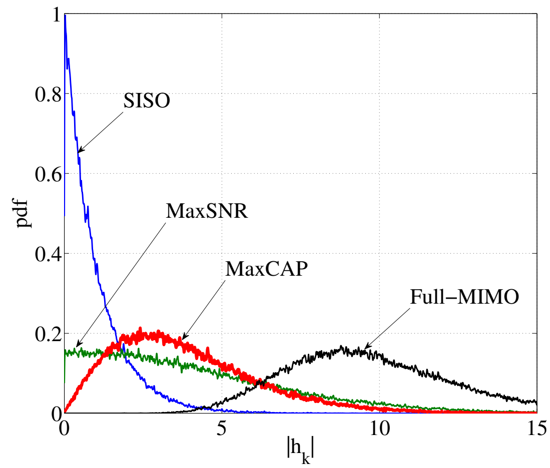


Fig. 3. Pdf of the equivalent channel response $|h_k|$. SNR=10 dB.

which is valid when $\gamma|h_k|^2 \ll 1$.

On the other hand, in the high SNR regime the mean square error rapidly decreases with the energy of the $k$-th channel $|h_k|^2$, which translates into different weights $\text{MSE}_k$ in the SIMO and MISO matrices (7) and (8). In this case, the proposed criterion tries to improve the spectral flatness of the equivalent SISO channel at the expense of the received SNR.

## V. Simulation Results

The performance of the proposed technique is illustrated in this section by means of some Monte Carlo simulations. In all the experiments, a Rayleigh MIMO channel model with exponential power delay profile has been assumed. In particular, we consider a $4 \times 4$ MIMO system and the total power associated to the $l$-th tap is

$$E\left[\|\mathbf{H}[l]\|^2\right] = \rho^l(1-\rho)n_T n_R, \qquad l = 0, \ldots, L_c - 1,$$

where we have selected $\rho = 0.7$ and $L_c = 16$. In all the experiments the number of subcarriers is $N_c = 64$ and we have used QPSK constellations, which can be either coded or uncoded. The proposed algorithm has never exceeded 50 iterations, and it has been compared with the MaxSNR approach proposed in [5], with a SISO system and with a full MIMO scheme applying maximum ratio transmission (MRT) and maximum ratio combining (MRC) per subcarrier (denoted as Full-MIMO), which can be seen as an upper bound for the performance of any analog antenna combining system.

### A. Analysis of the Equivalent Channel

In the first set of examples, we analyze the equivalent channel after beamforming. Fig. 2 shows the frequency response of the equivalent channel for one random channel realization and a SNR equal to 10 dB. We can see that, unlike the MaxSNR approach, the MaxCAP criterion avoids deep nulls in the frequency response of the equivalent channel. Furthermore, as expected, the performance of the MaxSNR and MaxCAP approaches is between that of the SISO and Full-MIMO system. Finally, Fig. 3 shows the probability density function (pdf) of the amplitude frequency response of the equivalent channel (averaged over the MIMO channel statistics), where it is clear that the MaxCAP criterion avoids values close to zero at the expense of a slight degradation of the overall SNR.

### B. Performance Analysis

The main advantage of the proposed criterion is that it achieves lower outage probabilities in comparison to the MaxSNR approach and the SISO system. This is illustrated in Fig. 4, which shows the outage probability for a transmission rate of 5 bps/Hz. As can be seen, the MaxCAP criterion obtains a gain of 2 dB over the MaxSNR approach for an outage probability of $10^{-4}$.

The development of an IC chip for the novel RF architecture based on the 802.11a standard is currently being pursued within the EU funded project MIMAX (*MIMO Systems for Maximum Reliability and Performance*), which has motivated us to evaluate the MaxCAP criterion with respect to this standard. Specifically, 48 out of 64 carriers are used for the transmission of data bits, which can be either uncoded or coded with a convolutional code of rate 1/2 followed by a block interleaver such as specified in the 802.11a standard. For both coded and uncoded transmission a QPSK modulation was used. For coded transmission the decoding process is based on a soft Viterbi decoder. The final results are shown in Figs. 5 and 6 for the cases of coded and uncoded transmissions, respectively. Firstly, we must note that the analog combining schemes incur in a degradation of the diversity order (slope of the BER-SNR curve) with respect to the Full-MIMO system. This degradation is due to the use of a common beamformer for all the subcarriers and it increases with the frequency selectivity of the channel.[4] However, the diversity order of the MaxCAP criterion is better than that of the MaxSNR beamforming, which in these examples is similar to that of

[4]Obviously, for a flat fading channel, the performance of both the MaxSNR and MaxCAP criteria is identical to that of the Full-MIMO system.

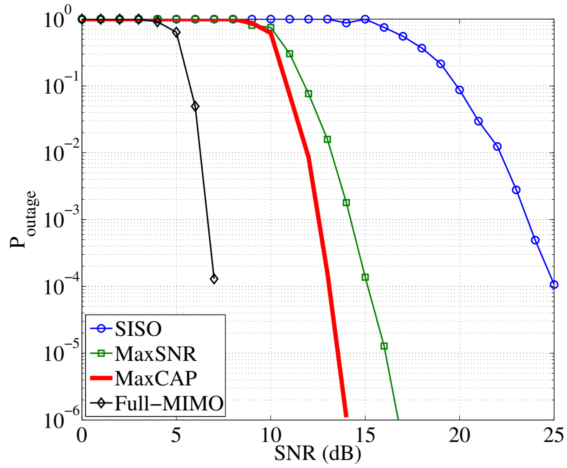


Fig. 4. Outage probability for a transmission rate of 5 bps/Hz.

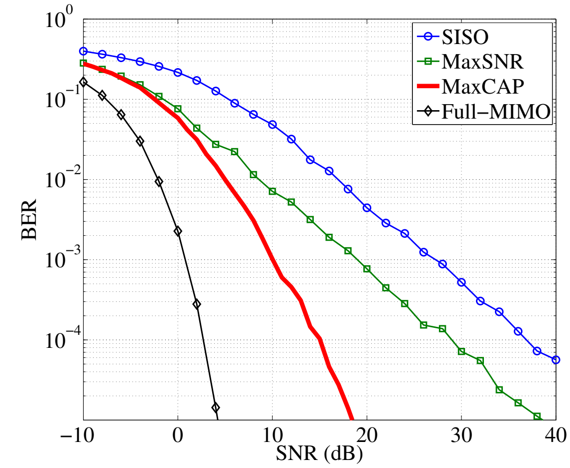


Fig. 6. BER performance. Uncoded transmissions.

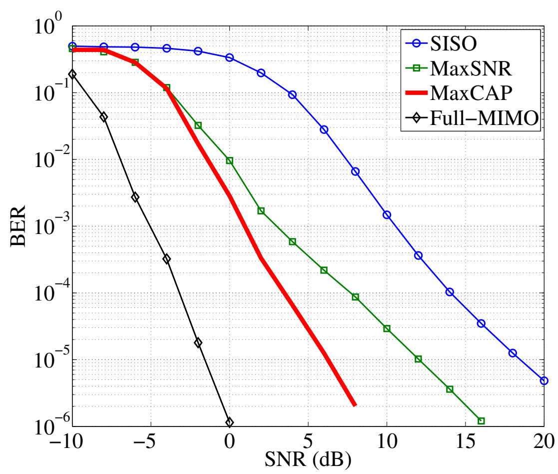


Fig. 5. BER performance. Coded transmissions.

a SISO system. In particular, the gap between the MaxCAP and MaxSNR approaches is very significant for uncoded transmissions: more than 14 dB for a BER of $10^{-4}$. This can be easily explained because for uncoded transmissions the BER is dominated by the worst subcarriers, which are improved by the MaxCAP criterion. On the other hand, although by using a channel encoder followed by an interleaver all the bits are transmitted through all the subcarriers (thus flattening somehow the channel), we see that the improvement of the worst subcarriers performed by the MaxCAP criterion is still very effective in reducing the BER: in this case the gain over the MaxSNR criterion is close to 3.5 dB for a BER of $10^{-4}$.

## VI. Conclusion

In this paper we have addressed the problem of adaptive antenna combining in the radio frequency (RF) path under multicarrier transmissions. In particular, a new technique for the selection of the transmit and receive beamformers (RF weights) has been proposed. The criterion consists of maximizing the capacity of the equivalent channel. Unlike previous approaches, which are typically based on the maximization of the received SNR, the proposed criterion takes into account the quality of all the flat-fading sub-channels, avoiding deep nulls which could have a strong impact on the final system performance. This improvement has been illustrated by means of some numerical examples using 802.11a coded and uncoded transmissions.

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